Noise Analysis and Comparison of Phase- and Frequency-Detecting Readout Systems: Application to SAW Delay Line Magnetic Field Sensor - Supplementary Material -

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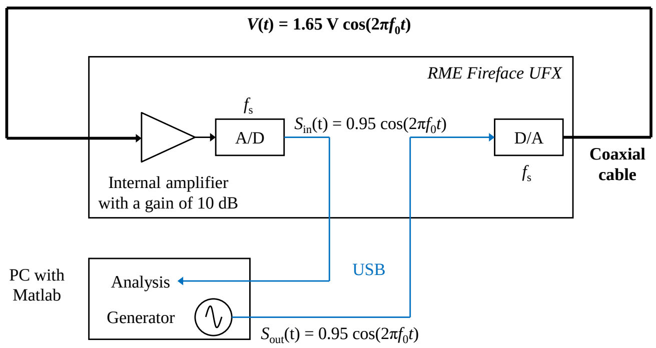

Fig. 1: Measurement setup for the characterization of the analog-to-digital converter *Fireface UFX* from *RME*.

INTRODUCTION

In the following, the 24-bit analog-to-digital (A/D) converter *Fireface UFX* from *RME* is characterized in terms of the spurious-free dynamic range (SFDR), the signal-to-noise-and-distortion ratio (SINAD), the effective number of bits (ENOB), and the signal-to-quantization-noise dynamic range (SQNDR). In the main article, the A/D converter is utilized for the digitization of both phase and frequency modulated sensor signals at an intermediate frequency of $f_0 = 50\,\mathrm{kHz}$.

MEASUREMENT

For the measurement a setup as depicted in Fig. 1 is used. A personal computer (PC) with running Matlab is used to numerically generate a harmonic signal

$$S_{\mathrm{out}}(t) = 0.95\cos(2\pi f_0 t) \tag{1}$$

with a frequency of $f_0 = 50\,\mathrm{kHz}$ and an amplitude of 0.95. Via a universal serial bus (USB) connection this unitless signal is transferred to the digital-to-analog (D/A) converter (channel 3 at the device's back side) which converts $S_{\mathrm{out}}(t)$ into an output voltage signal

$$V(t) = 1.65\,\mathrm{V}\cos(2\pi f_0 t) \tag{2}$$

with an amplitude of $1.65\,\mathrm{V}$. The amplitude of S_{out} is chosen slightly below full scale (amplitude of 1) in order to prevent unregular behavior. With a coaxial cable the D/A converter's output voltage signal $V(t)$ is directly fed back into the A/D converter (channel 9 at the device's front side). The A/D converter offers a gain-adjustable preamplifier which is set to a gain of $10\,\mathrm{dB}$ to ensure that the A/D converter is also driven slightly below full scale. Thus, the digitized signal

$$S_{\mathrm{in}}(t) = 0.95\cos(2\pi f_0 t) \tag{3}$$

has also an amplitude of 0.95 and is fed back to the PC for further analysis in Matlab. Both converters are running with a sample rate of $f_{\mathrm{s}} = 192\,\mathrm{kHz}$ and the measurement duration is $60\,\mathrm{s}$.

ANALYSIS

In Matlab, the power spectrum $\mathrm{PS}(f)$ of the digitized signal $S_{\mathrm{in}}(t)$ is calculated with Welch's method [1]

```
[PS, f] = pwelch(Sin, flattopwin(NFFT), [],
    NFFT, fs, 'power');
```

and utilizing a flat top window. From the power spectrum the linear spectrum

$$\mathrm{LS}(f) = \sqrt{\mathrm{PS}(f)} \tag{4}$$

is simply given by the square root of the power spectrum [2]. The length of the underlying fast Fourier transform (FFT) has been set to $N_{\mathrm{FFT}} = 5 f_{\mathrm{s}}/\mathrm{Hz}$. The equivalent noise bandwidth (ENBW) can be calculated by

$$\mathrm{ENBW} = f_{\mathrm{s}} \frac{\sum\limits_{n=0}^{N_{\mathrm{FFT}}-1} w^2(n)}{\left(\sum\limits_{n=0}^{N_{\mathrm{FFT}}-1} w(n)\right)^2} \tag{5}$$

where $w(n)$ is the window function (here flat top window) [2]. For the given parameters the ENBW results in a value of $\mathrm{ENBW} = 0.754\,\mathrm{Hz}$.

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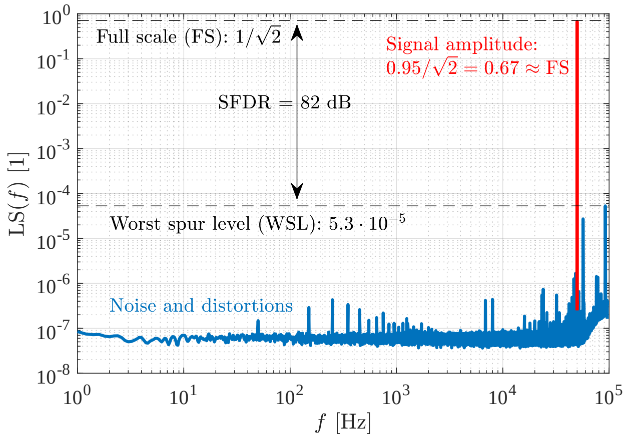
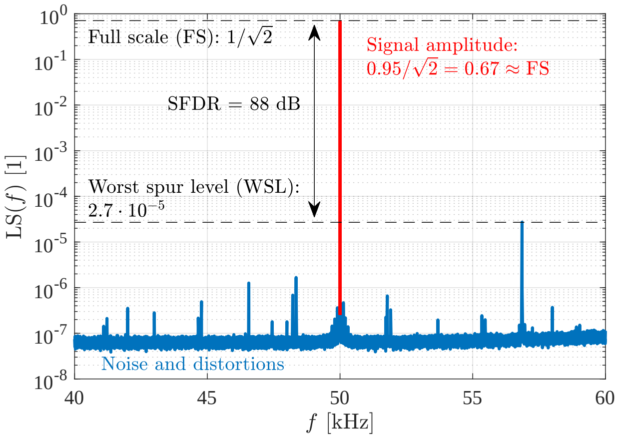
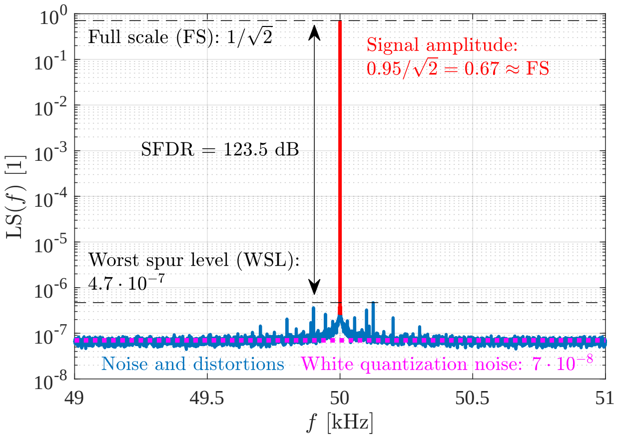


(a) Complete spectrum from 1 Hz to $f_\mathrm{s}/2$



(b) Spectrum for frequencies of ±10 kHz around f_0



(c) Spectrum for frequencies of ±1 kHz around f_0

Fig. 2: Linear spectrum $\mathrm{LS}(f)$ of the analog-to-digital converted signal $S_\mathrm{in}(t)$ for various frequency ranges.

Fig. 2 shows the resulting linear spectrum for various frequency spans which directly yield the spurious-free dynamic ranges [3, p. 93]. Due to large spurious signals the SFDR is only 82 dB when the complete spectrum is considered. For a frequency span of ±1 kHz around f_0 the SFDR increases to 123.5 dB. Assuming that the signal-to-noise-and-distortion ratio is dominated by the spurious signals, it is given by

TABLE I: Summary of measured and calculated parameters of the analog-to-digital converter *Fireface UFX* from *RME* for various frequency ranges.

f	1 Hz - $f_\mathrm{s}/2$	f_0±10 kHz	f_0±1 kHz
SFDR [dB]	82	88	123.5
LS(f_0)	0.67		
$\sqrt{\sum \mathrm{LS}^2(f)\|_{f>0\wedge f\neq f_0}}$	$155 \cdot 10^{-6}$	$60 \cdot 10^{-6}$	$7.5 \cdot 10^{-6}$
SINAD [dB]	72.8	81	99
ENOB [bit]	11.8	13.2	16.2

$$\mathrm{SINAD} = 20\log_{10}\left(\frac{\mathrm{LS}(f_0)}{\sqrt{\sum \mathrm{LS}^2(f)|_{f>0\wedge f\neq f_0}}}\right)\,\mathrm{dB} \quad (6)$$

the ratio of the RMS (root mean square) signal amplitude to the root-sum-squares (RSS) of all other spectral components excluding the spectral component at $f = 0\,\mathrm{Hz}$ [3, p. 92]. For each spectrum in Fig. 2 the signal $\mathrm{LS}(f_0)$ is plotted in red and the noise and the spurious signals $\mathrm{LS}(f)|_{f\neq f_0}$ are plotted in blue. With the SINAD, the effective number of bits can be determined by [3, p. 92]

$$\mathrm{ENOB} = \frac{\mathrm{SINAD} - 1.76\,\mathrm{dB}}{6.02\,\mathrm{dB}}\,\mathrm{bit}. \quad (7)$$

All measured and calculated values are summarized in Tab. I. Due to lower spurious signals in the smallest frequency range an effective number of bits of 16.2 is reached. However, as highlighted in Fig. 2c (magenta), the low white quantization noise density of

$$\mathrm{QN} = \frac{7 \cdot 10^{-8}}{\sqrt{\mathrm{ENBW}}} = 80.6 \cdot 10^{-9}\,\frac{1}{\sqrt{\mathrm{Hz}}} \quad (8)$$

results in a signal-to-quantization-noise dynamic range (SQNDR) of

$$\mathrm{SQNDR} = 20\log_{10}\left(\frac{\mathrm{LS}(f_0)}{\mathrm{QN}\cdot\sqrt{1\,\mathrm{Hz}}}\right)\,\mathrm{dB} = 138.4\,\mathrm{dB}. \quad (9)$$

This value is valid up to frequencies of $\approx 55\,\mathrm{kHz}$. For higher frequencies the quantization noise increases to a value of up to $\mathrm{QN} \approx 334 \cdot 10^{-9}\; 1/\sqrt{\mathrm{Hz}}$ at $f_\mathrm{s}/2$.

Finally, please note that all results might be impaired by the utilized D/A converter, i.e. the spurious signals might maybe stem from the D/A converter and are not produced inside the A/D converter.

REFERENCES

[1] P. Welch, "The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms," *IEEE Transactions on Audio and Electroacoustics*, vol. 15, no. 2, pp. 70–73, June 1967.

[2] G. Heinzel, A. Rüdiger, and R. Schilling, "Spectrum and spectral density estimation by the Discrete Fourier transform (DFT), including a comprehensive list of window functions and some new flat-top windows," Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Hannover, Deutschland, Tech. Rep., 2002.

[3] W. Kester, *The Data Conversion Handbook*. Amsterdam Boston: Elsevier Newnes, 2005, ISBN: 0-7506-7841-0, 2005.