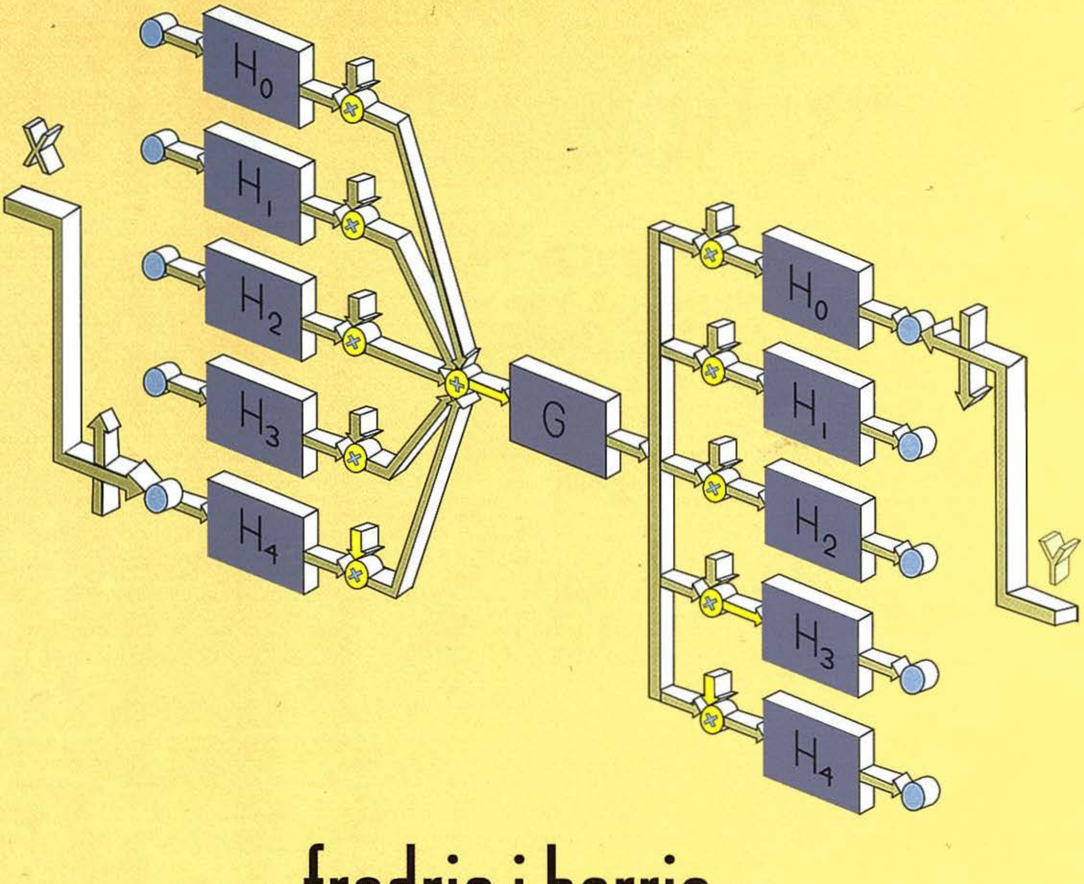
MULTIRATE SIGNAL PROCESSING
FOR COMMUNICATION SYSTEMS



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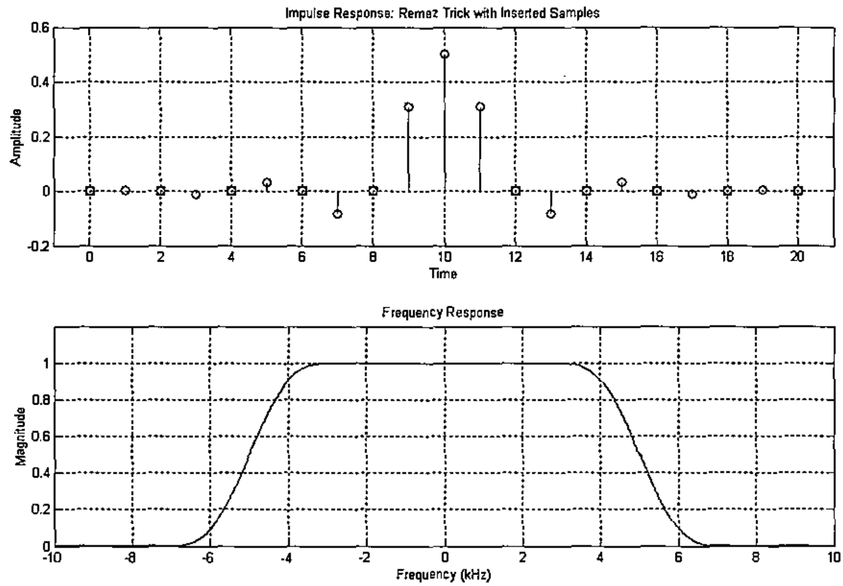

Figure 8.8 Impulse Response and Frequency Response of Trick One-Band Filter with Inserted Zero Samples and Symmetry Point Sample

8.5 HILBERT TRANSFORM BAND-PASS FILTER

The half-band filters described and designed in the previous sections have been low pass and high pass filters. A trivial variation of this design leads to a half-band filter that performs the Hilbert transform. The Hilbert transform has a frequency response with a nominal unity gain over the positive frequencies and nominal zero gain over the negative frequencies. The frequency response of the Hilbert transform form of the half-band filter is shown in Figure 8.9.

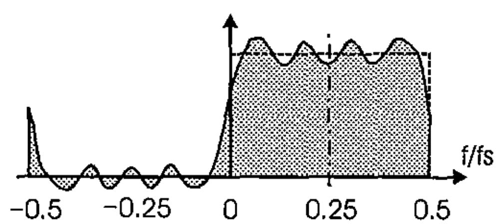

Figure 8.9 Spectral Characteristics of Hilbert Transform Half-band Filter

The spectrum of the Hilbert transform filter is obtained by translating the low pass half-band filter to the quarter-sample rate. This form of this translation for a noncausal filter is shown in (8.12).

$$h_{HT}(n) = h_{LP}(n)\, e^{jn\pi/2} : \; -N \leq n \leq +N \tag{8.12}$$

The heterodyne indicated in (8.12) could be visualized as a quadrature heterodyne with a cosine and a sine series as shown in (8.13).

$$h_{HT}(n) = h_{LP}(n)\, \{\cos(n\pi/2) + j\sin(n\pi/2)\} : \; -N \leq n \leq +N \tag{8.13}$$

We note that the cosine component of the heterodyne is a sequence comprised of unit amplitude samples with alternating signs on the even indices and zero-valued samples on the odd indices. The sequence is of the form $\{\;1\;\;0\;-1\;\;0\;\}$. The half-band filter is zero-valued at the even indices except for the sample at the origin, the symmetry point. Thus the cosine heterodyne, the real part of the heterodyne, contains a single non-zero sample at the origin. The sine component of the heterodyne is a sequence comprised of unit amplitude samples with alternating signs on the odd indices and zero-valued samples on the even indices. The sequence is of the form $\{\;0\;\;1\;\;0\;-1\;\}$. The odd indexed samples of the half-band filter contain the alternating sign side-lobe samples of the sinc(n π/2) series. The samples of the sine heterodyne interact with the samples of the sinc sequence to remove the alternating signs of the sequence. The complex impulse response and the corresponding spectrum are shown in Figure 8.10.

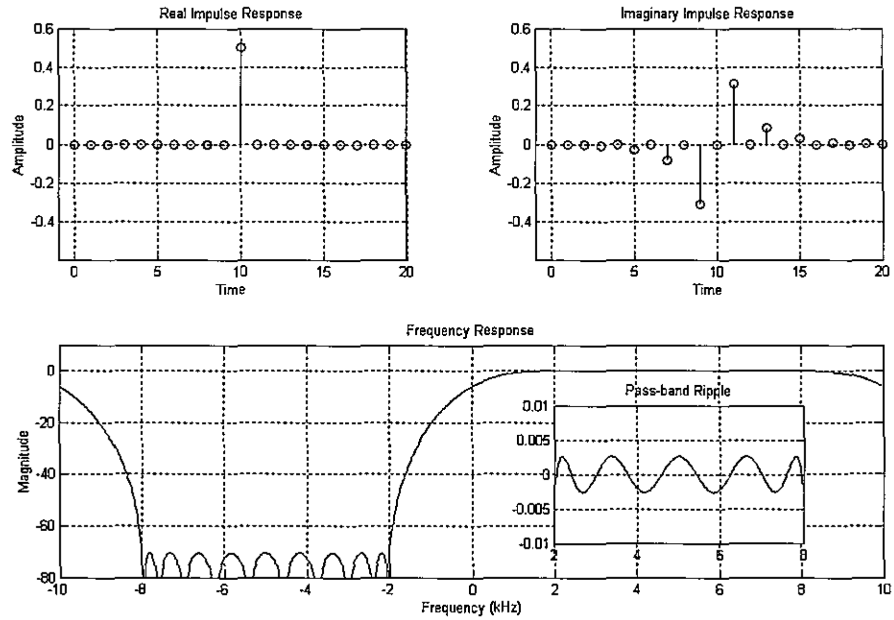

Figure 8.10 Impulse Response and Frequency Response of Half-band Filter Designed by Remez Algorithm and Converted to Hilbert Transform by Complex Heterodyne to fs/4

8.5.1 Applying the Hilbert Transform Filter

The discrete Hilbert transform filter can be used to generate a complex signal from a real input sequence by eliminating the spectral components residing in the negative frequency band. After suppressing the negative frequencies, thus reducing the bandwidth by a factor of 2, it is common to also reduce the sample rate by the same factor. Thus the Hilbert transform filter can be used as a half-band multirate filter. As an aside, this differs from the conventional Weaver modulator that translates the center of the positive frequency interval to baseband for processing by the half-band filter. The Hilbert transform filter translates the baseband half-band filter to the center of the positive frequency interval. This is the "which do we move, filter or signal spectrum" question we discussed in Chapter 6. Moving the filter to the spectral center is equivalent to moving the spectral center to the filter. Figure 8.11 illustrates spectral translation that forms the Hilbert transform filter.

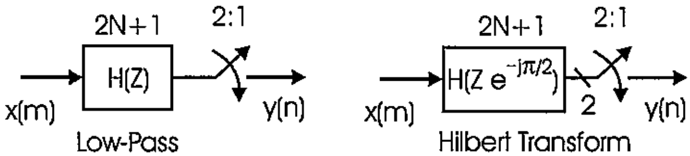

Figure 8.11 Hilbert Transform Filter as a Spectrally Translated Version of Low Pass Half-band Filter

The complex heterodyne of the low pass filter's impulse response results in a filter with complex impulse response. Such a filter can be formed as a two-path filter, one forming the real part and one forming the imaginary part of the impulse response. This form of the Hilbert transform filter is shown in Figure 8.12.

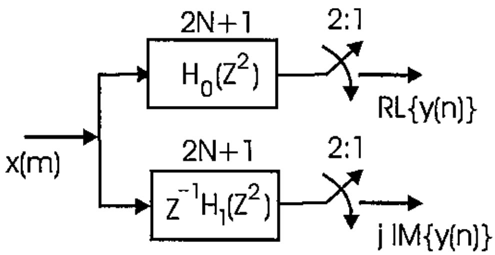

Figure 8.12 Two-path Model of Hilbert Transform Filter with Complex Impulse Response

Due to the zeros of the quarter sample rate cosine and sine, the impulse response of the upper path is seen to be zero at the even indices while the impulse response of the lower path is seen to be zero at the odd indices. The zeros permit application of the noble identity in which we interchange the filter with the 2-to-1 resample to operate the filters at the reduced output sample rate. This interchange is demonstrated in Figure 8.13.

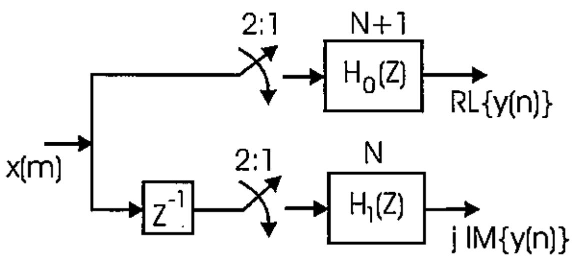

Figure 8.13 Noble Identity Applied to Hilbert Transform Filter

Finally, the interaction of the pair of 2-to-1 resampler switches and the input delay line can be replaced with a 2-input commutator that performs the same function. This form of the resampling half-band Hilbert transform filter is shown in Figure 8.14.

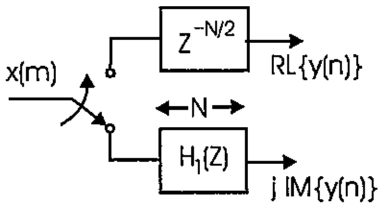

Figure 8.14 Two-path Commutator -driven Hilbert Transform Filter

8.6 INTERPOLATING WITH LOW PASS HALF-BAND FILTERS

The half-band low pass filter can be used to up sample a time series by a factor of two. The initial form of the1-to-2 up sampling process, based on zero insertion to raise the input sample rate, is shown in Figure 8.15.

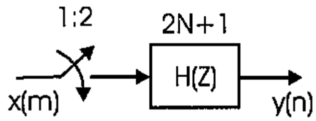

Figure 8.15 One-to-Two Up Sampling Process with Half-band Filter

The half-band filter H(Z) can be partitioned into a pair of polyphase filters as shown in (8.14) and (8.15) and illustrated in Figure 8.16.

$$H(Z) = \sum_{n=0}^{2N} h(n)\, Z^{-n} \tag{8.14}$$

$$H(Z) = \sum_{n=0}^{N} h(2n)\, Z^{-2n} + Z^{-1} \sum_{n=0}^{N-1} h(2n+1)\, Z^{-2n} \tag{8.15}$$