Passive and active RADAR using Software Defined Radio (SDR)

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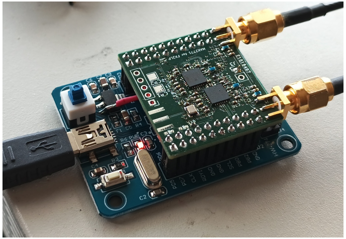

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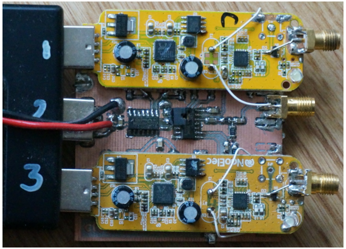

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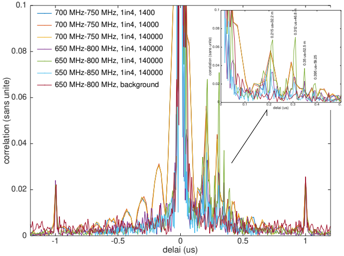
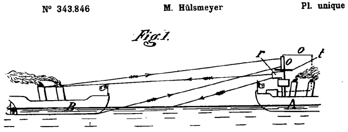


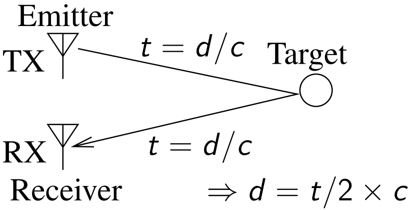
July 31, 2025

Definition of RAdiofrequency Detection And Ranging (RADAR [1] [2] [3])

- Electromagnetic waves propagate at a speed of 300 m/μs
- Electromagnetic waves are **reflected** by conducting or dielectric media
- The two-way **time of flight** t provides an estimate of the **range** d to the target
- Emitter and receiver at the same location: **monostatic** RADAR [a] [b] [c] [d]
- Emitter and receiver at separate locations: bi- or **multi-static** RADAR
- Target moving at velocity v induces (Doppler) frequency shift: $\delta f = \mathbf{2} \cdot f_c \frac{c}{v}$
- Static targets backscatter at the same frequency than incoming signal: **clutter**
- **Passive** RADAR: use surrounding electromagnetic smog for target ranging



C. Hülsmeyer (1904), *Télémobiloscope* patent application (in French) at https://patents.google.com/patent/FR343846A/en



[a] L. Brown, *A RADAR history of World War II – Technical and military imperatives*, CRC Press (1999)
[b] B. Lovell, *Echoes of War: The story of H2S RADAR*, CRC Press (1991)
[c] B. Johnson, *The secret war*, Methuen (1978)
[d] Y. Blanchard, *Le radar, 1904-2004 : Histoire d'un siècle d'innovations techniques et opérationnelles*, Ellipses (2004)

[1] T. Whipple, *The battle of the beams – the secret science of RADAR that turned the tide of WW2*, Random House (2023)
[2] R. Buderi, *The invention that changed the world – how a small group of RADAR pioneers won the second world war and launched a technological revolution*, Simon & Schuster (1996)
[3] D. Lewis, *Churchill's shadow raiders – the race to develop RADAR, WWII's invisible secret weapon*, Kensington Publishing Corp. (2019)

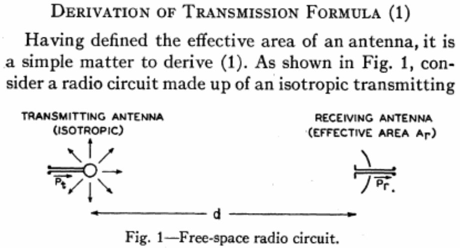
Signal requirements: power

Energy conservation from an isotropic transmitter to an isotropic receiver:

- emitted power P_E spreads over a sphere with area $4\pi \cdot d^2$ at distance d
- receiver with area λ^2 intersects the sphere
- received power P_R normalized with the 4π steradians surrounding the receiver

$$4\pi \cdot P_R = P_E \cdot \frac{\lambda^2}{4\pi \cdot d^2} \Leftrightarrow P_R = \left(\frac{\lambda}{4\pi \cdot d}\right)^2$$

- linear to decibel scale: $10\log_{10}()$: Free Space Propagation Loss
  $FSPL = 20\log_{10}(d) + 20\log_{10}(f) - 147.55$
  where $20\log_{10}(c/(4\pi)) = 147.55$ (d[m], f[Hz])
- Point like target acts as a new source: $1/d^2 \times 1/d^2 = 1/d^4$


DERIVATION OF TRANSMISSION FORMULA (1)

Having defined the effective area of an antenna, it is a simple matter to derive (1). As shown in Fig. 1, consider a radio circuit made up of an isotropic transmitting

Fig. 1—Free-space radio circuit.

H.T.Friis *A Note on a Simple Transmission Formula*, Proc.I.R.E. 254-–256 (1946)

RADAR Propagation Loss$= 40\log_{10}(d) + 40\log_{10}(f) - 147.55 \times 2$, decreasing as d^4

Signal requirements: bandwidth

- Monostatic RADAR range resolution: $\boxed{\Delta R = \frac{c}{2B}}$ with B the signal bandwidth ($\forall$ carrier frequency)
- The method used to reach B is irrelevant to the RADAR performance:

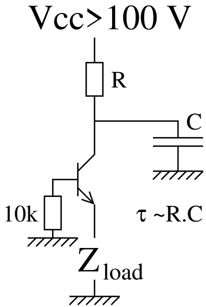
Method	Benefit	Drawback
Pulse	Time multiplexing of emission and reception	Energy concentrated in a short duration, challenging amplifier and pulse generator for SDR (e.g. avalanche transistor)
FMCW [4]	Beatnote receiver is audiofrequency	Linearity of the frequency swept source
FSCW	Narrowband IF filter for low integrated noise bandwidth (sensitive)	Probes each frequency one after the other
Noise	Probes all frequencies at the same time, stealth, **very suitable for SDR**	Heavy computational load

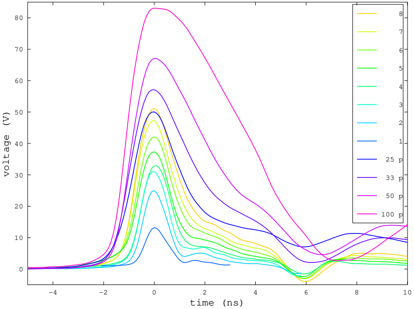
[4] G.L. Charvat, *Small and short-range radar systems*, CRC Press, 2014

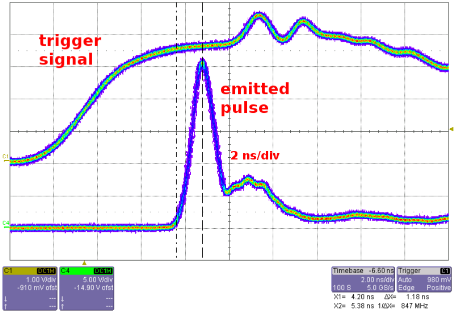
Signal requirements: bandwidth

- Monostatic RADAR range resolution: $\boxed{\Delta R = \frac{c}{2B}}$ with B the signal bandwidth (∀ carrier frequency)
- The method used to reach B is irrelevant to the RADAR performance:

Example of pulsed RADAR using an avalanche transistor as pulse transmitter and a **stroboscopic** [4] [5] (STM32F334 HRTIM, 217 ps step=1/4.6 GS/s equivalent) recording as receiver (used in Ground Penetrating RADAR).







[4] J. Williams, *High Speed Amplifier Techniques*, Linear Technology application note 47, pp.93–95 (1991)

[5] F. Minary, D. Rabus, G. Martin, J.-M. Friedt, *Note: a dual-chip stroboscopic pulsed RADAR for probing passive sensors*, Rev. Sci. Instrum. **87**, p.096104 (2016)

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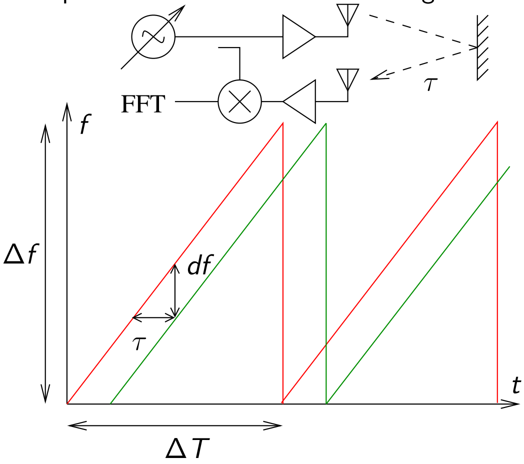
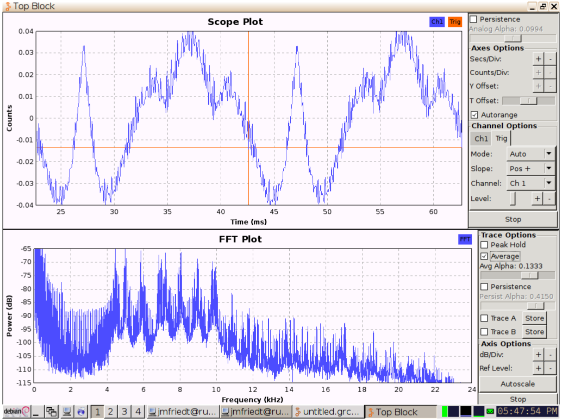
Example of FMCW measurement using a sound card: $\frac{df}{\tau} = \frac{\Delta f}{\Delta T} \Rightarrow df$ audiofrequency.





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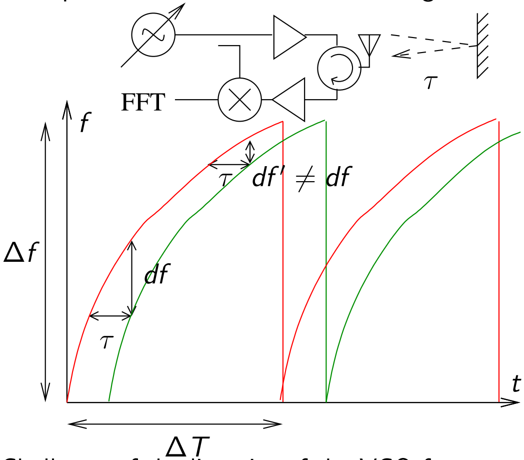

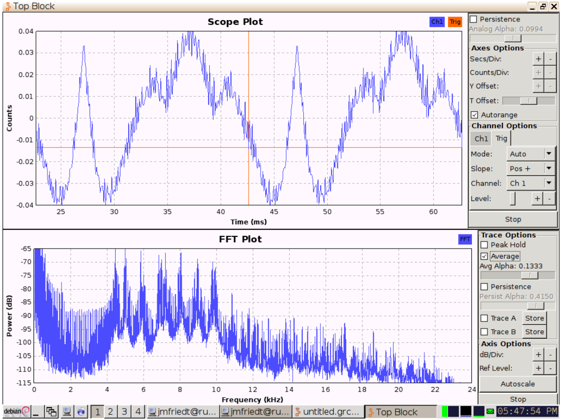

Challenge of the linearity of the VCO frequency output with time: $df' \neq df$ spreads the Fourier peaks

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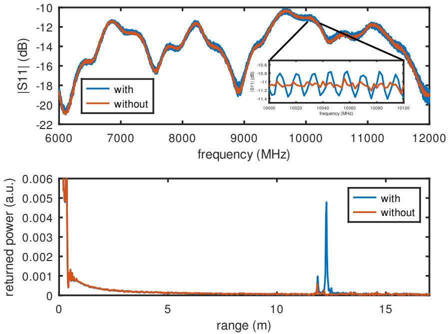
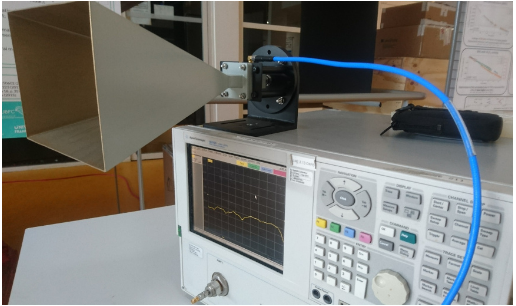
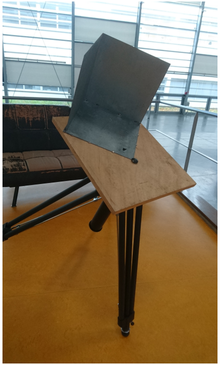
Timestep $1/B \Rightarrow PRI = N/B$ for N samples/measurement







Example of FSCW measurement using a commercial vector network analyzer [4].

[4] see NanoVNA – `https://nanovna.com/` – for low-cost embedded versions

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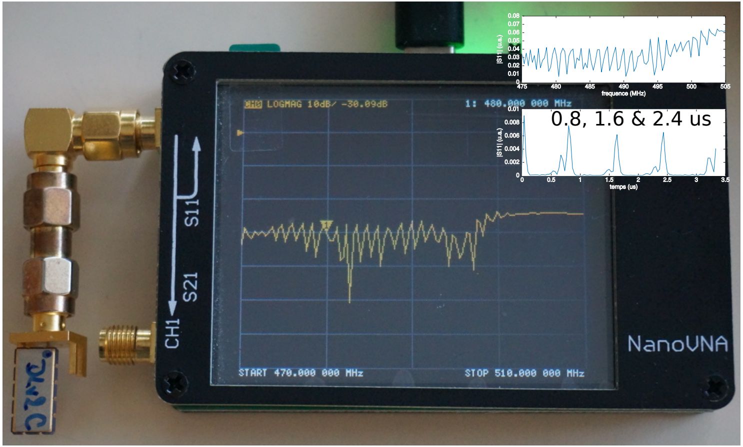

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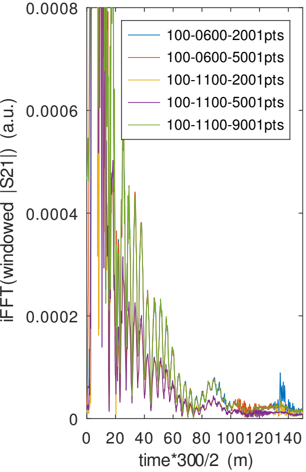
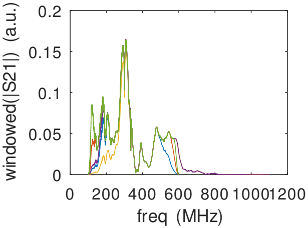
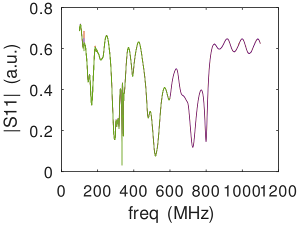






See LibreVNA – `https://github.com/jankae/LibreVNA` – for a practical broadband embedded version.

Signal requirements: correlation and ambiguity function

- Search for a pattern p in a signal s: cross-correlation

$$xcorr(s,p)(\tau) = \int s(t) \cdot p^*(t+\tau)dt \underbrace{\longrightarrow}_{\text{discrete time}} xcorr(s,p)_n = \sum s_k \cdot p^*_{k+n}$$

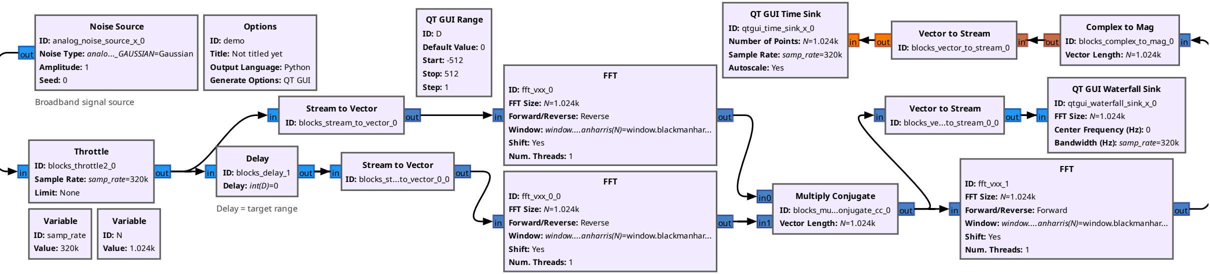
- When s includes p: matching capability determined by autocorrelation $xcorr(p,p)$
- Doppler shift for moving target: extend to the Ambiguity Function

$$AF(\tau,f) = \int p(t) \cdot p^*(t+\tau) \cdot \exp(j2\pi ft)dt$$

- Energy will not accumulate in the correlation peak if the **frequency shift between surveillance and reference** (s and p) bigger than a fraction of the inverse of the duration of p
- Computationally intensive operation: N multiplication for N values of $\tau \Rightarrow O(N^2)$ complexity

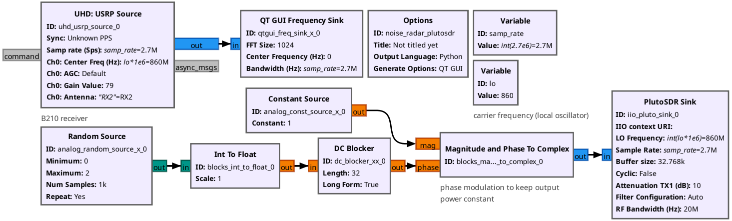
Correlation in the Fourier domain: demonstration using GNU Radio

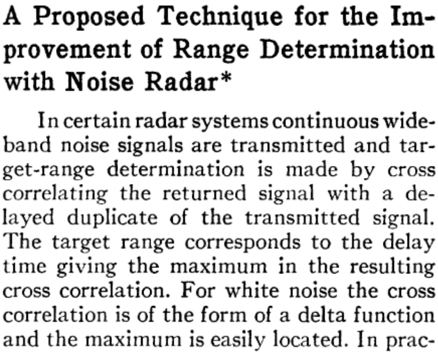
- Demonstration using GNU Radio... but no *xcorr*() block in GNU Radio
- computing the cross-correlation in the Fourier domain: $FT(xcorr(s,p)) = FT(s) \cdot FT^*(p)$
- Fourier transform in GNU Radio: Fast Fourier Transform FFT block ($O(N \ln(N))$)
- computing the integral $X(f) = \int x(t) \exp(j2\pi ft)$ requires accumulating enough samples: stream to vector
- Fourier transform is linear: $f(ax+by) = af(x) + bf(y) \Rightarrow$ multiple targets at different ranges will be individually detected even though the receiving antenna receives the sum of the backscattered signals



Monostatic active noise [4] [5] [6] RADAR demonstration

- PlutoSDR as noise source generator
- Record a minute fraction of the transmitted signal (coupler): reference channel
- Record the backscattered signal: surveillance channel
- Cross-correlate surveillance and reference





A Proposed Technique for the Improvement of Range Determination with Noise Radar*

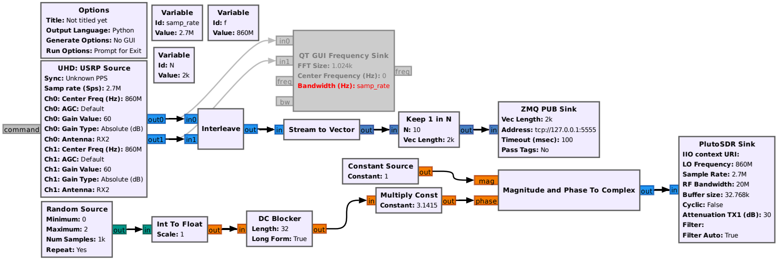
In certain radar systems continuous wide-band noise signals are transmitted and target-range determination is made by cross correlating the returned signal with a delayed duplicate of the transmitted signal. The target range corresponds to the delay time giving the maximum in the resulting cross correlation. For white noise the cross correlation is of the form of a delta function and the maximum is easily located. In prac-

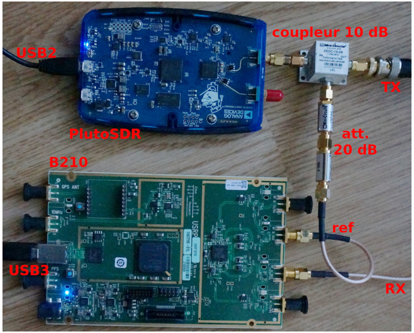
[4] R. Bourret, *A proposed technique for the improvement of range determination with noise radar*, Proc of IRE **45** (12) 1744–1744 (1957)

[5] B.M Horton, *Noise-Modulated Distance Measuring Systems*, Proc. IRE **47** (5), 821–828 (1959)

[6] K. A. Lukin & R. M. Narayanan, *Historical overview and current research on noise radar*, 3rd International Asia-Pacific Conference on Synthetic Aperture Radar (APSAR) (2011) cites a full volume of IET Radar, Sonar & Navigation (Aug. 2018) dedicated to noise RADAR at `ieeexplore.ieee.org/xpl/tocresult.jsp?isnumber=4607165`

Experimental setup





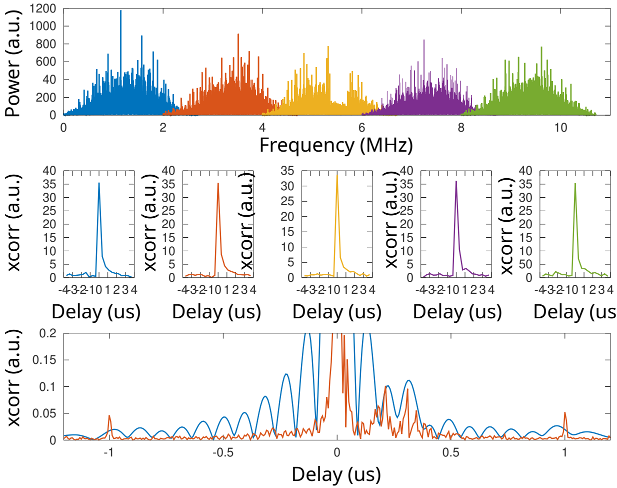
- Ettus Research B210 AD9361 RF frontend shares a common RX LO to both channels (coherent acquisition) [a]
- Datastream too fast for real time computation
- No need for continuous stream processing: grab a snapshot of broadcast signal and received signal for processing
- ZeroMQ Publish-Subscribe for non-blocking stream to C/Python/GNU Octave...

[a] but TX LO $\neq$ RX LO

Frequency stacking

- Transmitter bandwidth limited by communication bus datarate
- Cross-correlation calculation in the Fourier domain: accumulate spectra collected in adjacent frequency bands
- Spectrum stacking *requires a constant phase* over carrier scan duration
- Replace PlutoSDR transmitter with e.g. WiFi broadcast: using Bastian Bloessel's `packetspammer` at `https://github.com/bastibl/gr-ieee802-11`:

```
ifconfig wlan0 down
iwconfig wlan0 mode monitor
ifconfig wlan0 up
# loop on CHANNEL and collect @ each carrier
iwconfig wlan0 channel ${CHANNEL}
packetspammer wlan0 -n 1000000 -r 500 -s 1450
```



Passive RADAR[7]

- Hitchhike existing transmitters: powerfull ($1/d^4$) and broadband $\Rightarrow$ DVB-T transmitters
- Benefit from Doppler shift to separate moving targets from clutter (especially close to sea shore)

$$df = 2f_0\frac{v}{c}$$

- Computationally intensive Doppler-range maps: compute

$$RD(\tau, df) = \int sur(t) \cdot ref^*(t+\tau) \exp(j2\pi df \cdot t) dt$$

- Problem of phase stability and receiver synchronization during frequency stacking

[7] https://github.com/jmfriedt/passive_radar

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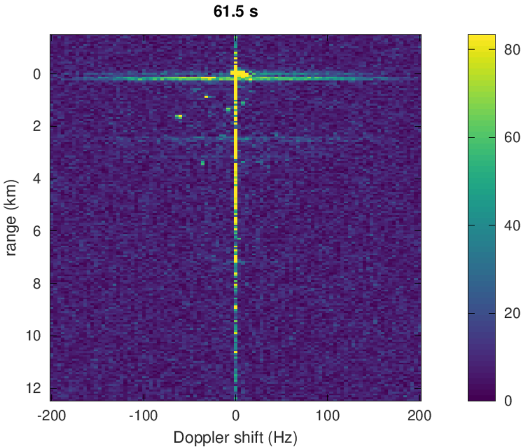

$f_0 \simeq 500$ MHz$\Rightarrow$ 20 Hz shift$=$ 22 km/h
Data collected using low-cost RTL-SDR receivers (2.048 MS/s)

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Vivaldi (left) v.s Yagi-Uda (right) surveillance antenna

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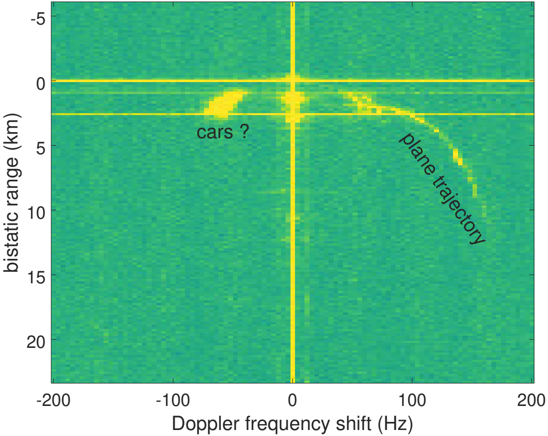
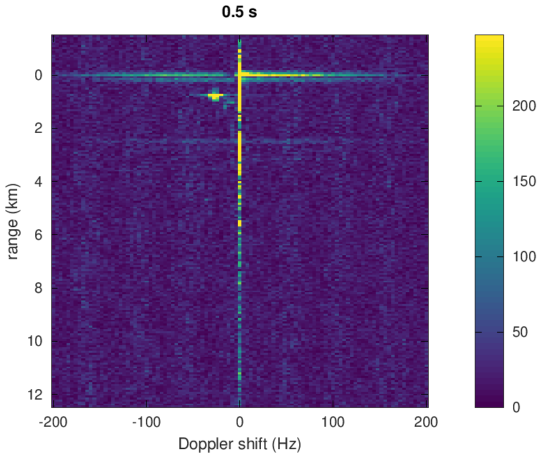
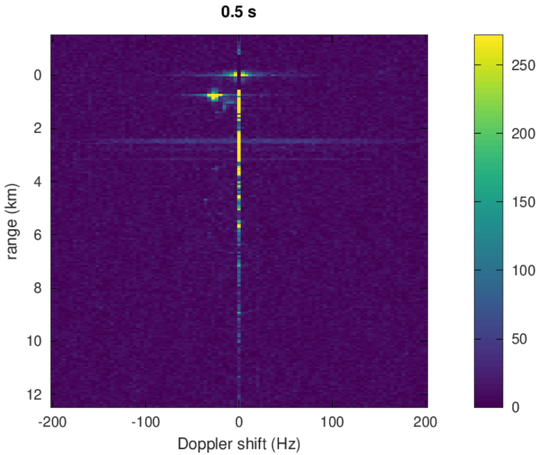


Data collected using low-cost RTL-SDR receivers (2.048 MS/s) [a]

[a] W. Feng, J.-M. Friedt, G. Cherniak, M. Sato, *Passive bistatic radar using digital video broadcasting–terrestrial receivers as general-purpose software-defined radio receivers*, Rev. Sci. Instrum **89**(10) 104701 (2018)





Left: no Direct Signal Interference (DSI) removal

Right: with DSI removal

Direct Signal Interference (DSI) and its removal

- energy leaked from the reference channel into the surveillance channel
- might create unwanted artifacts close to 0-delay if autocorrelation is not a sharp Dirac function
- search for delayed copies of the reference signal in the surveillance: least square optimization problem

$$sur = W \cdot refs$$

  with W vector the weights of the delayed copies of ref

$$W = sur \cdot refs^{-1}$$

- create a matrix $refs$ with time delayed copies of the reference: $refs$ is not square so what is $refs^{-1}$?
- Penrose (1955) & al: pseudo-inverse $M^{-1} = (M^t \cdot M)^{-1} \cdot M^t$
- $\Rightarrow W = sur \cdot (refs^t \cdot refs)^{-1} \cdot refs^t = sur \cdot pinv(refs)$
- $sur \leftarrow sur - W \cdot refs$ to subtract delayed copies of ref from sur

$$refs = \begin{bmatrix} 0 & 0 & 0 & r_1 & r_2 & r_3 & r_4 \\ 0 & 0 & r_1 & r_2 & r_3 & r_4 & \ldots \\ 0 & r_1 & r_2 & r_3 & r_4 & \ldots & r_{N-1} \\ r_1 & r_2 & r_3 & \ldots & \ldots & r_{N-1} & r_N \\ r_2 & \ldots & \ldots & \ldots & r_{N-1} & r_N & 0 \\ \ldots & r_{N-3} & r_{N-2} & r_{N-1} & r_N & 0 & 0 \\ r_{N-3} & r_{N-2} & r_{N-1} & r_N & 0 & 0 & 0 \end{bmatrix}$$

many more rows (samples) than columns (delays)

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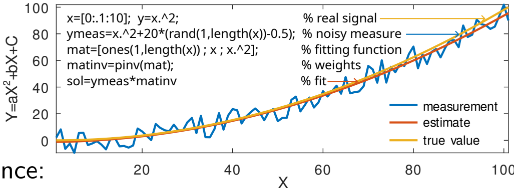

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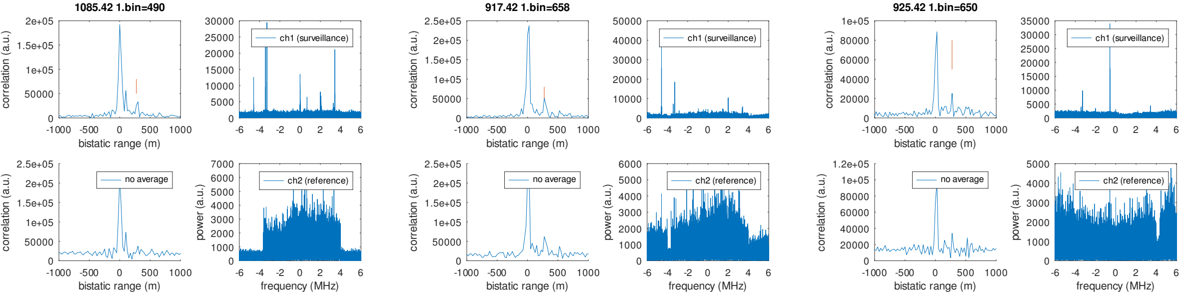
Multistatic RADAR range analysis

- Known transmitter-target-receiver delay, but the possible positions of the targets matching this observation is not so easy as a circle when transmitter and receiver are not co-located.
- Solution is an ellipse with the emitter and receiver locations as focal points...
- ... and the two-way trip as one of the points on the ellipse



- Data collected using (PocketSDR) dual-MAX2771 receivers (2 bit ADC but up to 44 MS/s) [8] with external frequency transposition to reach GNSS L1 band
- Vertical line at 913 ns i.e. (300 m/μs) 274 m=2 $\times$ 137 m but where is the target?

[8] https://github.com/jmfriedt/max2771_fx2lp/tree/main/250608_no_preamp

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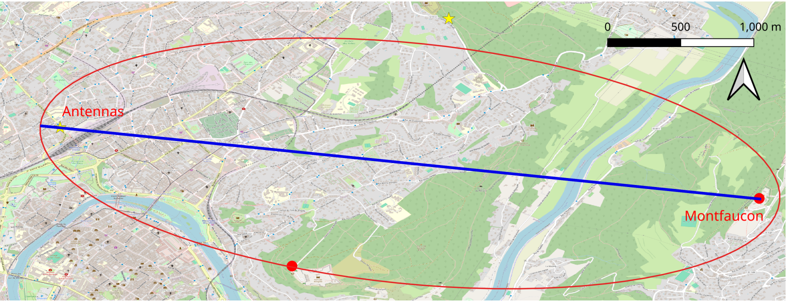
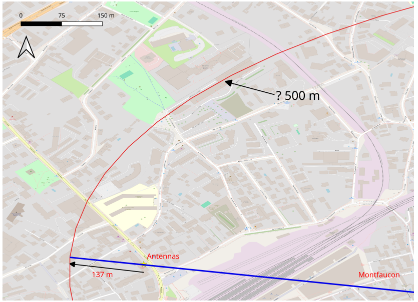
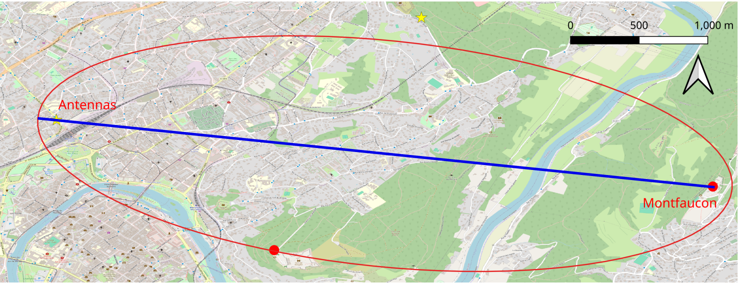
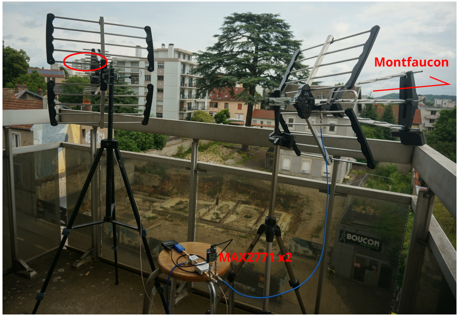




Multistatic RADAR range analysis

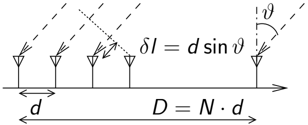
- Known transmitter-target-receiver delay, but the possible positions of the targets matching this observation is not so easy as a circle when transmitter and receiver are not co-located.
- Solution is an ellipse with the emitter and receiver locations as focal points...
- ... and the two-way trip as one of the points on the ellipse





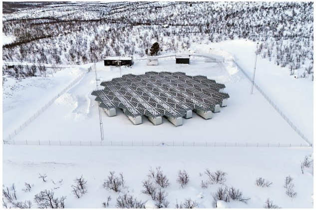
Adding azimuth resolution

- Complement range measurement with direction of arrival for {azimuth,range} mapping
- Requires a narrow beam ⇒ large antenna (diameter D) since beamwidth $\simeq \lambda/D$
- Impractical ⇒ synthesize an equivalent antenna by multiplying emitter/receivers along baseline with length D
- MISO (Multiple Inputs Single Output)/MIMO (Multiple Inputs Multiple Outputs)[a] well suited but challenging due to phase calibration of each receiving/emitting element



- Move a single emitter/receiver at different locations (constant phase offset at all positions): Sythetic Aperture RADAR (SAR)

[a] https://hforsten.com/mimo-radar-antenna-arrays.html

EISCAT → EISCAT 3D


Maritime communication VSATs

https://eiscat.se/wp-content/uploads/2024/02/PET_in_place.jpg
https://eiscat.se/wp-content/uploads/2016/05/Kiruna_site-1920x1280.jpg
https://www.chinastarwin.com/uploads/15203/starwin-sw020-maritime-antenna123fd.png
https://www.jonsa.com.tw/0-9m-ku-band

Synthetic aperture RADAR (SAR): Ground-Based ... (SDR-GB-SAR)

- Directional antenna requires aperture $\gg \lambda$ (hardly practical)
- Adding **azimuth resolution** with spatial diversity: Shannon sampling theorem in the spatial domain (dependent on the carrier frequency): move antenna with step δp:

$$\delta p \leq \lambda/4$$

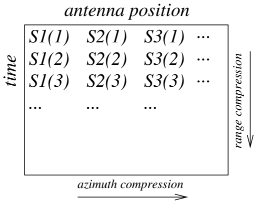
when moving TX & RX or

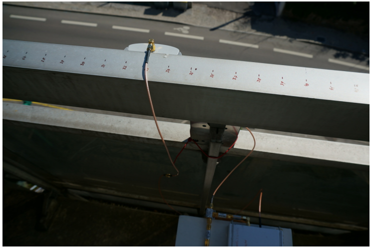
$$\delta p \leq \lambda/2$$

when moving TX **or** RX

- ZMQ data streaming for uninterrupted local oscillators but only grabbing measurements as antennas are static

- 2.45 GHz WiFi $\Rightarrow \frac{\lambda}{2} = \frac{300}{2\times 2450} = 6.12$ cm
- aperture $\simeq 2$ m $\Rightarrow 200/6.12 \simeq 32$ steps
- phase increment along antenna displacement: $d\varphi = 2\pi\frac{\delta p}{\lambda}\sin\vartheta$
- azimuth compression: Fourier transform along the antenna displacement axis $\Rightarrow$ accumulate energy in $\sin\vartheta$ bin





SAR signal processing:

1. for each antenna position $N \times \delta p$, collect a time domain vector from reference and surveillance $ref(t)$, $sur(t)$
2. Fourier transform each time domain vector and multiply conjugate: $FT(sur) \cdot FT^*(ref)$
3. 2D iFT of the resulting matrix for range-azimuth compression: $(p, f) \underbrace{\rightarrow}_{iFT} (\sin\vartheta, \underbrace{c\tau}_{r})$
4. project $(\sin\vartheta, range)$ to polar coordinates $(\vartheta, range)$

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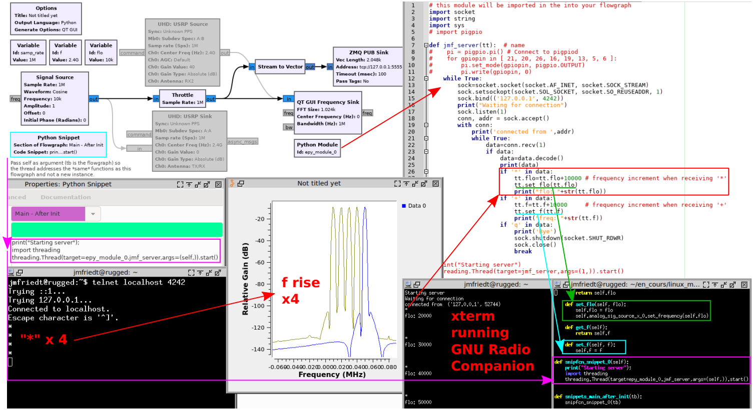

- ZeroMQ Publish-Subscribe ($\simeq$ UDP broadcasting) transfer of data from GNU Radio
- TCP/IP server thread (Python module) for controlling the antenna position/emitter-receiver frequency...
- ... launched from a Python snippet.
- Octave/Python client for collecting data, processing and displaying (remotely)

Synthetic aperture RADAR (SAR): Ground-Based ... (SDR-GB-SAR)

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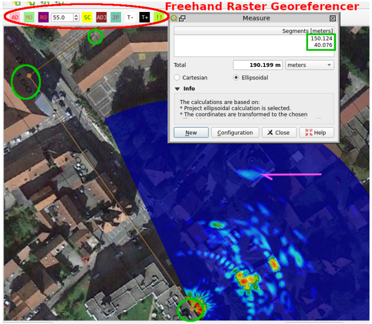
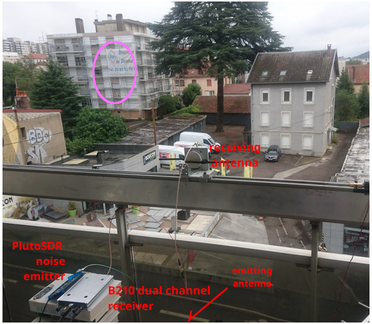
  when moving TX **or** RX
- ZMQ data streaming for uninterrupted local oscillators but only grabbing measurements as antennas are static
- Using WiFi 5.8 GHz frequency band for "smaller" antennas





Use of QGis Freehand Raster Georeferencer to import $(\theta, range)$ maps: position $(0, 0)$ on RADAR location, scale to match X and Y distances, and adjust orientation to match targets

Synthetic aperture RADAR (SAR): Ground-Based ... (SDR-GB-SAR)

- Directional antenna requires aperture $\gg \lambda$ (hardly practical)
- Adding **azimuth resolution** with spatial diversity: Shannon sampling theorem in the spatial domain (dependent on the carrier frequency): move antenna with step δp:

$$\delta p \leq \lambda/4$$

when moving TX & RX or

$$\delta p \leq \lambda/2$$

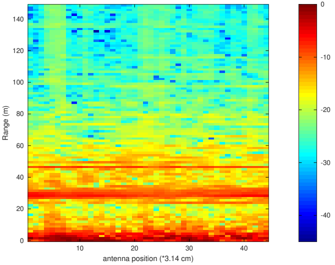
when moving TX **or** RX

- ZMQ data streaming for uninterrupted local oscillators but only grabbing measurements as antennas are static

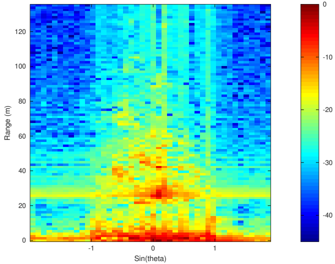
No degree of freedom in mapping the targets once positioning step and frequency bandwidth is determined, other than SDR-GB-SAR location and azimuth orientation

Software Defined Radio-Ground Based-Synthetic Aperture RADAR

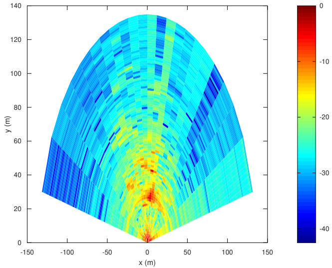
Check acquisition and processing step by step:



- Antenna position along the X-axis
- Cross-correlation along the Y-axis ($iFFT(FFT(sur) \cdot FFT^*(ref))$ or $iFFT(FFT(sur)/FFT(ref))$ to cancel amplitude variations during frequency stacking).



- Azimuth compression with iFFT along the X-axis



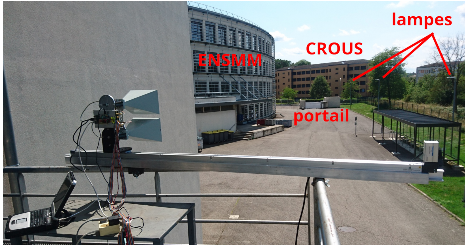

```
Nf = 100    % number of frequencies
Na = 73     % number of ant. pos,
c = 3e8     % speed of light
f = 2.45e9  % center frequency
df = 1e6;   % freq. sweep step
lambda = c/f
dx = lambda/4;
fs_r = 1/df;
r = (0:Nf-1)*fs_r/Nf*c/2;
fs_a = 1/dx;
alpha = (0:Na-1)*fs_a/Na-fs_a/2;
sin_t=alpha*lambda/2;

[R,ST]=meshgrid(r,sin_t(abs(sin_t)<=1));
X = R.*ST;Y = R.*sqrt(1-ST.^2);
Z=img(:, (abs(sin_t) <= 1));
pcolor(X.', Y.', 10*log10(Z));
```

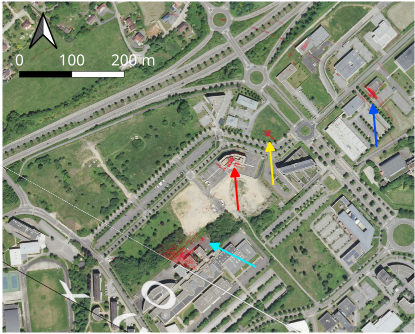
Courtesy of W. Feng

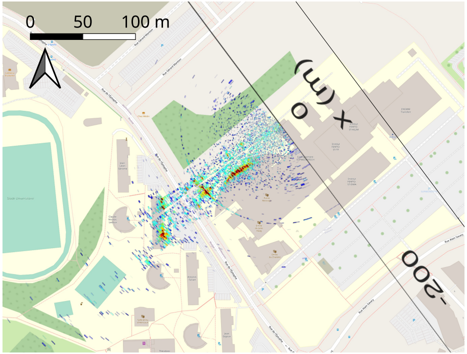
Software Defined Radio-Ground Based-Synthetic Aperture RADAR

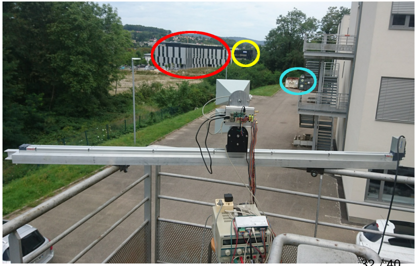
- 5 GHz WiFi emitter as pseudo-random signal source
- SDR sampling at 5 MS/s each frequency band controlled by a Raspberry Pi4
- frequency stacking prior to correlation calculation by sweeping the carrier from 5480 to 5720 MHz
- 250 MHz bandwidth ⇒ 60 cm range resolution
- targets visible to a 500 m range (850 samples)
- narrow beam due to high gain horn antenna
- $\simeq$ 20 min/data collection
- 20 W total power consumption

https://github.com/jmfriedt/SDR-GB-SAR



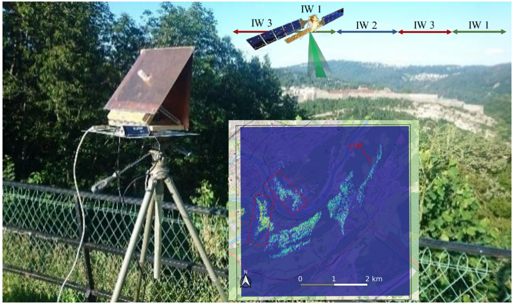


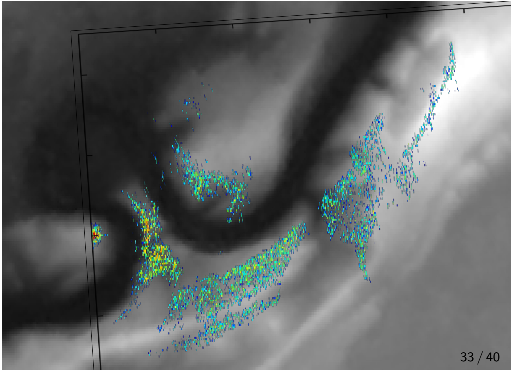




Passive Synthetic aperture RADAR (SAR)

- Moving TX/fixed RX: Sentinel1 spaceborne C-band RADAR [a]
- Worth generalizing to the many spaceborne signals now available (communication satellites, RADAR satellites: P-band Biomass, L and S-band NISAR, Ku-band ICEYE-X2[b])
- P-band: 435 ± 3 MHz
- L-band: 1257 ± 40 MHz (also for GNSS)
- S-band: 3200 ± 37.5 MHz
- C-band: 5405 ± 50 MHz (also military G-band RADARs)
- Ku-band: 9650 ± 75 MHz
- ... determine the azimuth resolution and penetration in foliage/snow/ice...
- spaceborne DVB-S receiver LNB are 9.75 GHz LO to transfer Ku to VHF/UHF (Starlink 11325 MHz beacon to 1575 MHz, in the GNSS L1 band!)

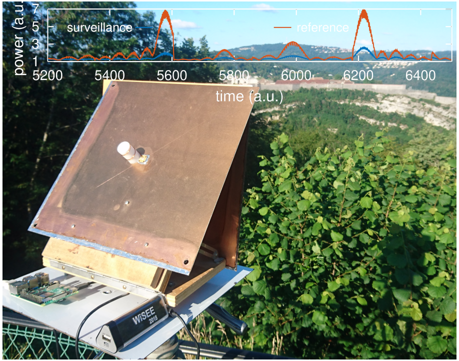




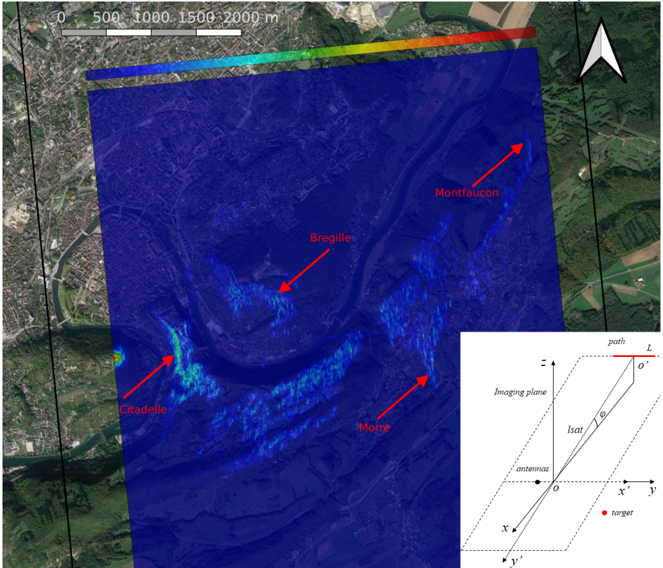
[a] W. Feng, J.-M. Friedt, P. Wan, *SDR-implemented passive bistatic SAR system using Sentinel-1 signal and its experiment results*, MDPI Remote Sensing **14**(1) pp.221– (2022)

[b] B. Falęcki & al., *Passive Bistatic SAR Imaging Using ICEYE-X2 Satellite as an Illuminator of Opportunity: First Experimental Results*, Proc. IEEE RADAR Conference (2025)

Passive bi-static measurement using Sentinel1 spaceborne RADAR as non-cooperative source (`github.com/jmfriedt/sentinel1_pbr`)

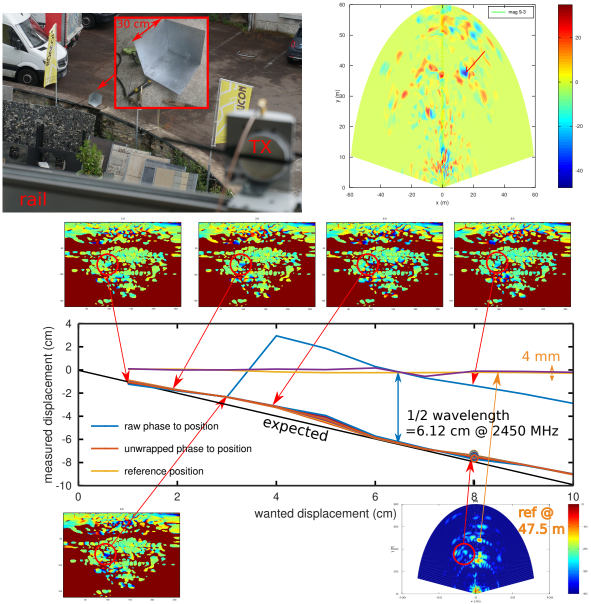


- no need for accurate satellite position computation: only satellite velocity is needed with pulse repetition interval
- assumption that satellite-target distance ≫ target-receiver distance
- accurate scale and positioning of reflectors over aerial image ⟶



Sub-wavelength motion detection: using the phase of the correlation

- $$d\varphi = \frac{2\times 2\pi \cdot dr}{\lambda} \Leftrightarrow dr = \frac{d\varphi \cdot c(T,P,RH)}{4\pi f}$$
- Dependent on atmospheric conditions: requires one reference fixed target to assess the impact of temperature T, pressure P and moisture RH variations on time of flight
- Sentinel1 spaceborne RADAR [a]: repeat pass of 12 days for each satellite [b], 6 days for A/C ⇒ InSAR analysis for fine displacement measurement using natural or artificial scatterers

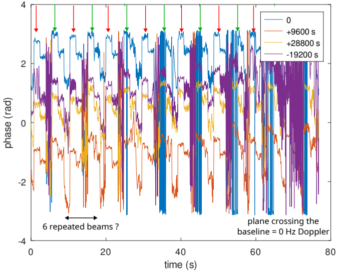


[a] https://github.com/jmfriedt/sentinel1_level0

[b] raw and processed data @ https://dataspace.copernicus.eu/browser

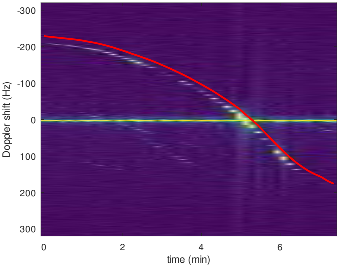
Fixed receiver but moving target: ISAR

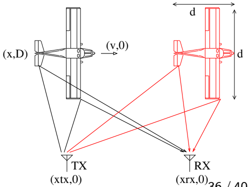
- GRAVES emitter ($f_0 = 143.05$ MHz)
- CW wave, electronic beam steering every 1.6 s



- planes ⇒ Doppler shift $\delta f = 2 \cdot f_0 \frac{v}{c} \simeq v$ [m/s]
- ±300 Hz frequency offset but no range resolution
- antenna array for azimuth/elevation: direction of arrival + projected speed ⇒ position
- plane motion along path: closest and furthest wing tips do not move at the same speed ⇒ fine-Doppler analysis after removing generation motion induced frequency offset: **Inverse SAR** (ISAR)

GRAVES emitter located 40 km from Besancçon





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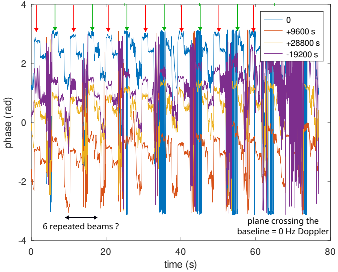

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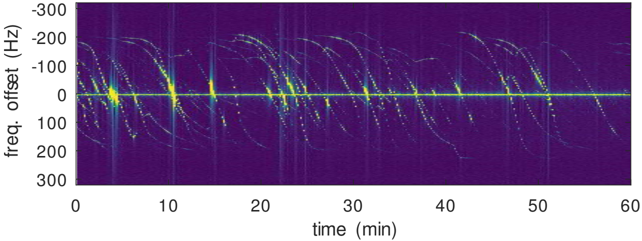
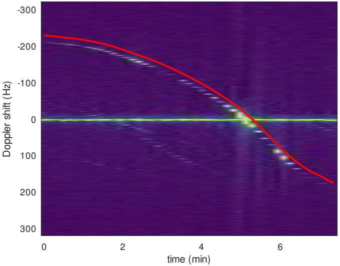
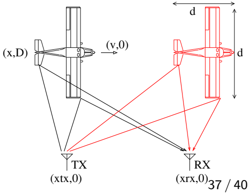
Direct signal is straight horizontal line: SDR oscillator disciplined on a hydrogen maser







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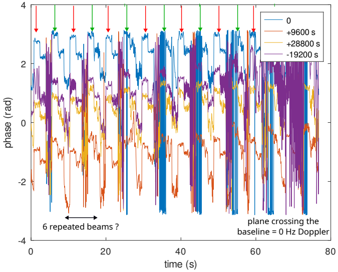

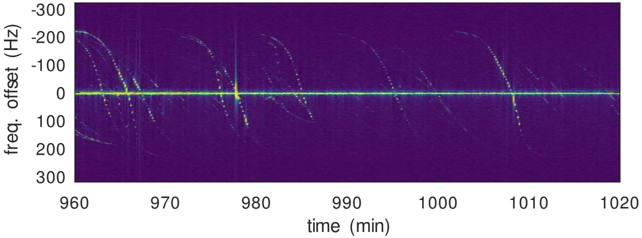

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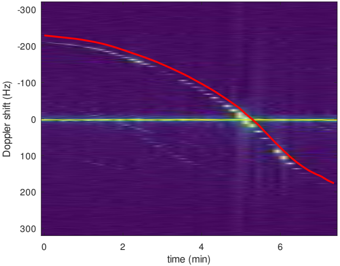

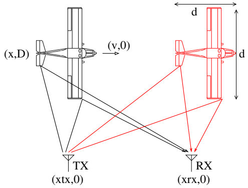

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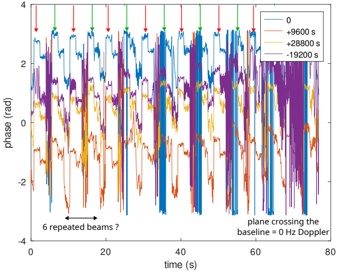

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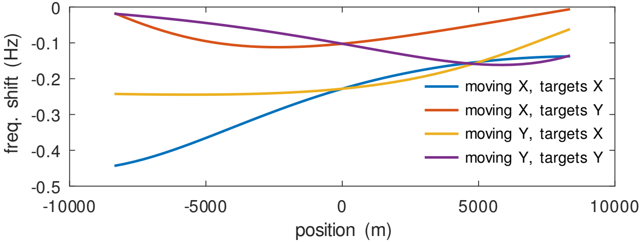
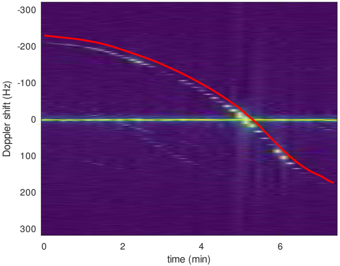
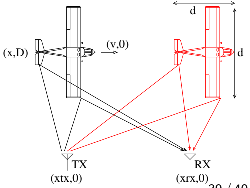
Simulated Doppler shift depending on plane path v.s TX/RX orientation







Conclusion

Experimenting with RADAR using SDR:

- all signal processing concepts related to RADAR signal analysis simulated with **GNU Radio** ...
- ... using flowcharts readily compatible with samples collected from real hardware (RTL-SDR, B210, PlutoSDR, oscilloscope)
- Many more applications available, including radio-astronomy, GNSS [a] signal processing, GNSS jamming and spoofing mitigation.
- **Heterogeneous** processing framework: collect and pre-process using GNU Radio, ZMQ stream to remote/offline processing (GNU Octave, Python) compatible with **embedded** systems (Raspberry Pi): GNU Radio provided with Buildroot ("never compile on the target")
- **Passive** RADAR benefits from the many signals broadcast in the radiofrequency bands without requiring a license for emitting.

More information?: GNU Radio Conference (GRCon), FOSDEM SDR devroom, Software Defined Radio Academy (SDRA), European GNU Radio Days

Acknowledgements: W. Feng (Air Force Engineering University, Xi'an, China), M. Sato (CNEAS, Tohoku University, Sendai, Japan)

[a] Global Navigation Satellite Systems positioning using time-flight measurement of spaceborne atomic clock signals and trilateration.