Bulk acoustic resonator characterization

A quartz bulk acoustic resonator as used for defining microcontroller clock rates for example has been characterized in the frequency and time domain. The objective is to process the collected data to extract the relevant characteristics. All files are available at `http://jmfriedt.free.fr/exam_M2acoustic_2021.tar.gz`

1 Bulk acoustic wave velocity calculation

The resonator under study is made of a thin slice of AT-cut quartz (rotational symbol (YXl)-35.25°), with metal electrodes deposited on the top and bottom sides. It operates on a thickness shear mode.

The elastic constants for quartz are usually given, as in the slowness calculation tutorial, by :

$$c = \begin{pmatrix} 0.867 & 0.070 & 0.119 & -0.179 & 0 & 0 \\ 0.070 & 0.867 & 0.119 & 0.179 & 0 & 0 \\ 0.119 & 0.119 & 1.070 & 0 & 0 & 0 \\ -0.179 & 0.179 & 0 & 0.579 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.579 & -0.179 \\ 0 & 0 & 0 & 0 & -0179 & 0.3985 \end{pmatrix}$$

The elastic constant tensor for AT-cut quartz is given as:

$$e = \begin{pmatrix} 0.867 & -0.082 & 0.271 & -0.037 & 0 & 0 \\ -0.082 & 1.297 & -0.074 & 0.058 & 0 & 0 \\ 0.271 & -0.074 & 1.029 & 0.099 & 0 & 0 \\ -0.037 & 0.058 & 0.099 & 0.386 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.688 & 0.025 \\ 0 & 0 & 0 & 0 & 0.025 & 0.290 \end{pmatrix},$$

the density ρ is equal to 2.65 g.cm^{-3}, the piezoelectric tensor reads:

$$c = \begin{pmatrix} 0.171 & -0.153 & -0.018 & 0.067 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.108 & -0.095 \\ 0 & 0 & 0 & 0 & -0.076 & 0.067 \end{pmatrix}$$

and the permittivity tensor is given by:

$$\epsilon = \begin{pmatrix} 3.920 & 0 & 0 \\ 0 & 3.981 & 0.086 \\ 0 & 0.086 & 4.042 \end{pmatrix}$$

1. Without running the calculation, give an account of the method you would have used to obtain the elastic constants for AT-cut quartz from the elastic constants of quartz.

2. We now consider a propagation along the [001] axis of the AT-cut quartz subtrate.
   (a) Let us first neglect the piezoelectricity of the material. Determine the velocity of the three bulk waves expected to propagate in the medium. Describe the equations and methods used to solve this problem.
   (b) The resonator operates on the highest velocity shear mode. What should be the plate thickness to allow for a fundamental mode at 25 MHz? Comment on the technological feasibility.
   (c) We now take the piezoelectricity into account. Which velocity component(s) is (are) going to be affected by the piezoelectricity of the material? Please justify.

2 Frequency domain characterization

Three frequency domain measurements have been collected using a vector network analyzer and saved in Touchstone S1P format.

3. The first file `10K.s1p` was collected with an intermediate bandwidth of 10 kHz, meaning that the duration between two successive acquisitions is 100 μs. Plot the magnitude of the S_{11} scattering parameter and comment on the aberration you observe on the plot. Can you explain a possible cause of the inconsistency?

4. The measurement was repeated with a 100 Hz intermediate frequency, meaning that the network analyzer waits for 10 ms between two successive measurement. The resulting dataset is found in `100.s1p`. Plot the magnitude of the S_{11} scattering parameter and comment how the result compares with the previous analysis. What is the frequency of the fundamental mode of this device? What is the second visible mode on the plot, and how does its frequency relate to the former?

5. The resolution of the initial broadband characterization is insufficient to extract the parameters of the fundamental mode. A zoom on the frequency axis is provided in `100Z.s1p`.

   (a) What is the operating frequency of this device?

   (b) What is the motional resistance (resistance in the Butterworth-VanDyke model)?

   (c) What is its quality factor?

   (d) The electromechanical coupling coefficient K^2 is given for a bulk acoustic device by $K^2 = \frac{\pi}{2}\frac{f_r}{f_a}\cot\left(\frac{\pi}{2}\frac{f_r}{f_a}\right)$ which is approximated for weakly coupled materials by its Taylor expansion as $K^2 \simeq \frac{\pi^2}{4}\left(\frac{f_a-f_r}{f_a}\right)$ with f_r the resonance frequency at the maximum of the admittance and f_a the antiresonance at the maximum of the impedance. Does the resulting value match your expectation for quartz? How does the result you find compare with the literature, e.g. C.S. Lam, *A Review of the Recent Development of Temperature Stable Cuts of Quartz for SAW Applications*, Proc. Fourth International Symposium on Acoustic Wave Devices for Future Mobile Communication Systems (2010) at `https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.623.1568&rep=rep1&type=pdf`?

6. From the answer to section 1 and this analysis, what is the thickness of the quartz plate propagating the shear wave?

3 Time domain characterization

M. Rodahl & al have observed the time domain decay of a Bulk Acoustic Resonator for characterizing a relevant quantity affected by biomolecule conformations and more generally the interaction of bulk acoustic waves with viscous media. An example of such a measurement on the same resonator than evaluated previously is provided in the file `3_24MHz9995.Wfm.csv` with the parameters found in `3_24MHz9995.csv`, most significantly the sampling period of 5 ns.

7. Analyze the collected data: how is the resonator frequency deduced from this measurement, and with what resolution? How does it compare with the resonance width?

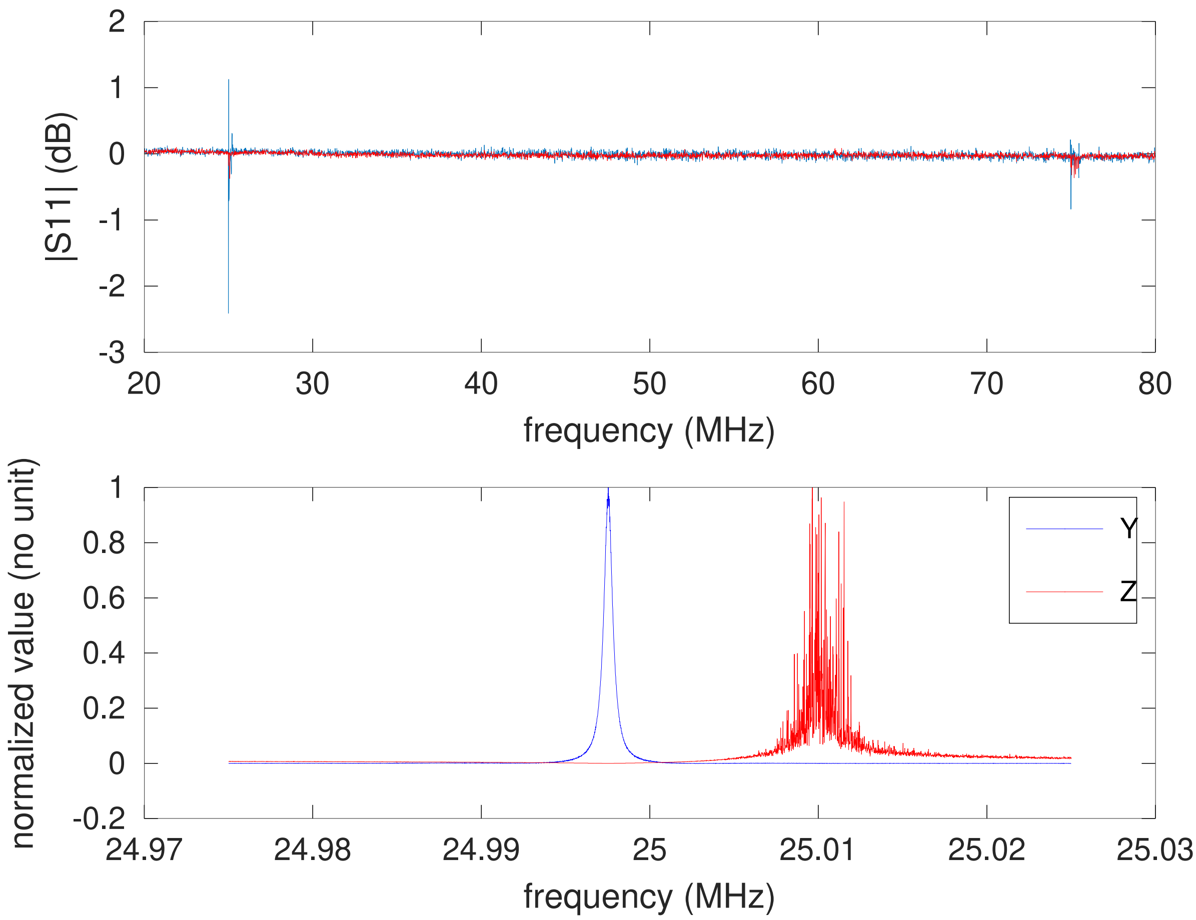
8. How is the curve shape related to damping? How is damping related to the quality factor? How can the collected data be displayed so its evolution appears linear with time?

9. From such a chart with a linear evolution of the envelope over time, deduce the quantity representative to molecule conformation or viscous interaction of the wave with its environment.

Answers:

```
clear all
close all

figure(1)
x=dlmread('10K.s1p','',8,0);
s11=x(:,2)+j*x(:,3);
f=x(:,1);
subplot(211)
plot(f/1E6,20*log10(abs(s11)))

x=dlmread('100.s1p','',8,0);
s11=x(:,2)+j*x(:,3);
f=x(:,1);
hold on
plot(f/1E6,20*log10(abs(s11)),'r')
xlabel('frequency (MHz)')
ylabel('|S11| (dB)')

figure(2)
y11=(1-s11)./(1+s11);
plot(f/1E6,real(y11),'r')

figure(1)
```

```
x=dlmread('100Z.s1p','',8,0);
s11=x(:,2)+j*x(:,3);
y11=1/50*(1-s11)./(1+s11);
z11=1./y11;
f=x(:,1);
subplot(212)
plot(f/1E6,real(y11)/max(real(y11)),'b')
hold on
plot(f/1E6,abs(z11)/max(abs(z11)),'r')
xlabel('frequency (MHz)')
ylabel('normalized value (no unit)')
legend('Y','Z')
yy=real(y11)/max(real(y11));
k=find(yy>0.5);df=f(k(end))-f(k(1));Q=25e6/df

x=dlmread('RefCurve_2021-10-29_2_195958.Wfm.csv',',');
Ts=5e-9;
figure
subplot(211)
plot([0:length(x)-1]*Ts,20*log10(1e-10+abs(x(1:end))))
ylim([-40 0])
xlabel('time (s)')
ylabel('power (dB)')
```



3. despite a correct calibration, the reflection scattering parameter of the passive sensor (no energy source that would allow the scattering parameter to rise above 0 dB) becomes positive due to ringing during more time than the duration spent between each measurement. The resonator has stored energy that is released during the next frequency measurement, leading the instrument to measure a higher returned voltage than injected.

4. Reducing the IF bandwidth increases scan duration and prevents ringing. Fundamental mode at 25 MHz fundamental mode, overtone 3 at 75 MHz

5. $f_r \simeq 24.9975$ MHz as the maximum of the admittance, with a width at half height of the admittance of 550 Hz or a Q of about 44000. The antiresonance is $f_a \simeq 25.0101$ MHz or $K^2 \simeq 0.13\%$, very close to the value provided in the article. The maximum of the real part of the admittance, when its imaginary part vanishes, is the inverse of the motional resistance of about 50 Ω.

6. The plate thickness is half the wavelength of a 25 MHz resonator propagating a bulk shear wave at 5060 m/s or 100 μm.

7. The Fourier transform spanning from $-fs/2$ to $fs/2$ with $fs = 1/Ts$ with $Ts = 5$ ns the sampling period displays a single peak at the resonance frequency. The duration of the ringdown is about 3 ms so that the frequency is measured with a 300 Hz resolution or about the width at half height. The exponential decay of the voltage V due to viscous dissipation of the acoustic wave follows a $\exp(-t/\tau)$ law with $\tau = Q/(\pi f_r)$ proportional to the quality factor which is inversely proportional to damping or viscous losses ...

8. ... so that $20\log_{10}(V) = -20\log_{10}(e) \times t/\tau$ so that the linear slope of the dB display of the voltage is representative of $\tau = Q/(\pi f_r)$. Measuring a 30 dB variation over 2.88 ms hints at $Q \simeq 66000$ which is close enough to the previous estimate.

9. When a narrowband signal is analyzed, aliasing or stroboscopic measurements allow for analyzing radiofrequency devices with slow analog to digital converters by introducing a strong assumption on the Nyquist zone the signal lies in. This is the technique used by Rodahl & al. to collect the ringdown signal of the bulk acoustic resonator. at