

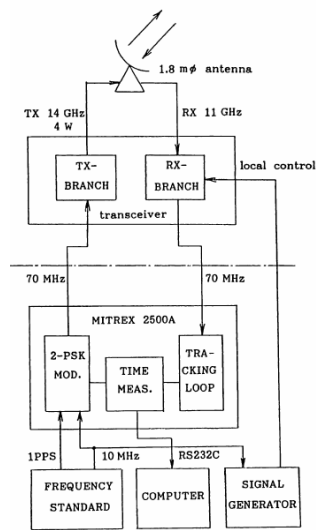
# Digital communication exam

J.-M Friedt, April 14, 2022

Every even hour of every day, metrology laboratories around the world share information about their atomic clocks through radiofrequency links broadcast through geostationary satellites located 36000 km above the equator. The signal is Binary Phase Shift Keying (BPSK) modulated and streams at a rate of 2.5 MS/s. The Time and Frequency department of FEMTO-ST has installed a 2.4 m parabola dish (Fig. 1) to receive and transmit such a signal to partner laboratories including PTB in Germany, OP in Paris, INRIM in Torino or the National Physics Laboratory (NPL) in London. Below is an excerpt of an article published on this two-way link, the two way communication between the laboratories allowing to assess the time of flight of the electromagnetic waves between emitter and receiver and hence remove this common contribution in order to deduce the time offset between remote clocks, with sub-ns second accuracy. Our objective is to assess whether such a signal can be recorded using a commercial, off the shelf 60 cm diameter parabola dish as sold in consumer electronic stores for receiving Digital Video Broadcast through Satellite (DVB-S) television signals.



**Figure 1:** 2.4 m-diameter parabola reflector antenna at FEMTO-ST for Two Way Satellite Time and Frequency Transfer



**Fig. 5** Schematic diagram of the CRL system.

**Table 2** Link budget

Transponder in high-gain mode	
Earth station transmit E.I.R.P.	52 dBW
Uplink pass loss	207 dB
Uplink tracking loss	1 dB
Satellite G/T at beam edge	0 dB/K
Uplink C/T	-156 dBW/K
Gain of 1 m <sup>2</sup> antenna	45 dBi/m <sup>2</sup>
Power flux density arriving at satellite	-111 dBW/m <sup>2</sup>
Transponder saturation flux density toward the earth station	-79 dBW/m <sup>2</sup>
Input back-off	32 dB
Output back-off	26 dB
Total transponder saturation E.I.R.P.	43 dB
Downlink E.I.R.P.	17 dBW
Downlink pass loss	205 dB
Downlink tracking loss	1 dB
Earth station G/T	19 dB/K
Downlink C/T	170 dB/K
Total C/N <sub>0</sub>	59 dB-Hz

**Telstar 11N Antenna Configuration**  
**Figure 2**

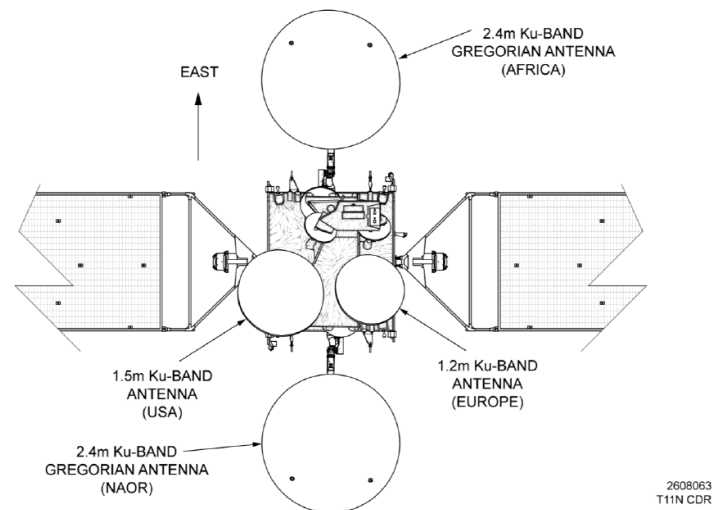


Figure 2: Left and middle: ground station configuration and link budget analysis from [1]. Right: Telstar 11N configuration [2], with Ku band referring to the 10.7–12.75 GHz downlink and 12–18 GHz uplink. E.I.R.P means “Equivalent Isotropically Radiated Power” and accounts for the antenna gain. “NAOR” refers to North Atlantic Ocean Region.

1. What is the wavelength of the transmitted signal? of the received signal?
2. For each wavelength, what is the parabola antenna gain on the ground? on the satellite?
3. Considering the signal transmitted from ground carrier frequency and the distance to the satellite assumed to be located at the same latitude than the broadcast stations, and a parabola reflector efficiency of 66%, what is the power reaching the satellite? Explain the calculation you made and compare with some of the entries in Fig. 2, not all of which are relevant to this calculation.
4. The satellite receiver must avoid saturation. We are told by the satellite manufacturer that the power density at the satellite must not exceed  $-97$  dBW/m<sup>2</sup>. How can you adapt the Free Space Propagation Loss (or Friis equation) to compute this spatial energy density? Is this condition met with your link budget?
5. Once the signal has been received, the satellite broadcasts the same signal, transposed to the new downlink frequency, amplified at the nominal power which in our case is a radiated power of +17 dBW (EIRP meaning Equivalent Isotropic Radiated Power, i.e. the power that would be radiated by an isotropic antenna). What was the electrical power feeding the antenna needed to radiate 17 dBW on this satellite configuration? How does it compare with the 100 W amplifier rating on-board the satellite?

Notice that I now realize that the geostationary satellite does not broadcast a fixed power but will amplify the received signal with a fixed gain of about 100 dB

6. Both the emitted and received signals are focused through the same parabolic dish, and the two way time of flight provides an accurate estimate of the station to satellite distance. What is the power received from the satellite at the ground station?
7. What is the thermal noise level on the 5 MHz bandwidth containing the signal? Compare this level with the received power. Can the signal be seen on a spectrum analyzer?
8. We now replace the 2.4 m dish with the consumer television reception dish. Repeat the last question – considering the downlink only – and answer the same question in the latter case.

Samples recorded from a Software Defined Radio connected to a 2.4 m parabola dish pointing towards Telstar 11N are provided in the file <http://jmfriedt.org/telstar.bin>. The samples have been recorded at 5 MSamples/s, 16 bit resolution, and are stored as interleaved IQ as short integers. The content of the file can be read either with:

- `f=fopen('telstar.bin');x=fread(f,inf,'int16');` with GNU Octave or...
  - using the File Source block from GNU Radio and setting the data type Short (yellow) followed by the iShort to Complex block to interleave I and Q, or...
  - using Python `import numpy as np;x=np.fromfile('telstar.bin',dtype=np.int16)`
9. what is the duration (in seconds) of the record?
  10. display the spectrum of the signal computed on 32768 bins so that the thermal noise in each bin is low enough. Can you observe any feature? If yes describe these features, if not why?
  11. Each communication channel is slightly offset from its nominal carrier frequency by up to 50 kHz: identify the frequency offset of the transmitting channels by displaying the spectrum as before, but this time after the appropriate processing to accumulate the energy in the carrier by de-spreading the spectrum, i.e. cancelling the BPSK modulation.
    - (a) Describe the processing steps and provide a few frequency offsets.
    - (b) How did you zoom in the spectrum around the area of interest?
    - (c) How many stations are communicating?
  12. Since the low-noise block downconverter (LNB) reception head which converts the 10-12 GHz microwave signal to a radiofrequency signal less attenuated by the cable reaching the television or Software Defined Radio receiver must be powered with 13 V at a current of about 100 mA, and yet only a single cable runs from the receiver to the satellite dish, what circuit would you use to power the active elements (amplifier, local oscillator, mixer) at the focal point of the parabola dish without preventing the radiofrequency signal from reaching the receiver? Provide a schematic of what this circuit would look like, with typical passive component values selected wisely according a demonstration of the impact of these components on the signal distribution.
  13. Similar to GPS, the communication protocol uses a known pseudo random sequence which has here been selected to be 10000 bit long. What is the impact of this knowledge on the signal to noise ratio? What mathematical operation allows for recovering the signal to noise improvement? Quantify this improvement?
  14. Will the signal become visible even with the small 60 cm parabola?

### Questions

15. What is the spectral resolution of a 2048 sample Fourier transform of a signal sampled at 1.024 Msamples/s?
16. Describe all the features of the spectrum of a 5 kHz sine wave used to AM modulate with modulation depth of 50% a 100 MHz carrier.
17. What is the typical bandwidth of a commercial FM broadcast station?
18. What is Doppler shift introduced by a car driving at 100 km/h towards the emitter of a commercial FM broadcast station?
19. How do these two quantities compare? Is the FM reception affected by the motion of the vehicle? What is the consequence of the motion on the FM reception chain and the signal feeding the loudspeaker?
20. In the low signal to noise ratio condition as seen for the geostationary satellite communication link, what quantity between the emitted power and transmitted bandwidth drives the channel capacity?
21. Is an unlicensed user allowed to transmit at 137.5 MHz? at 434 MHz? Justify your answer.
22. How long does it take for an electromagnetic signal to propagate along a 900 m path in air? Justify the answer.

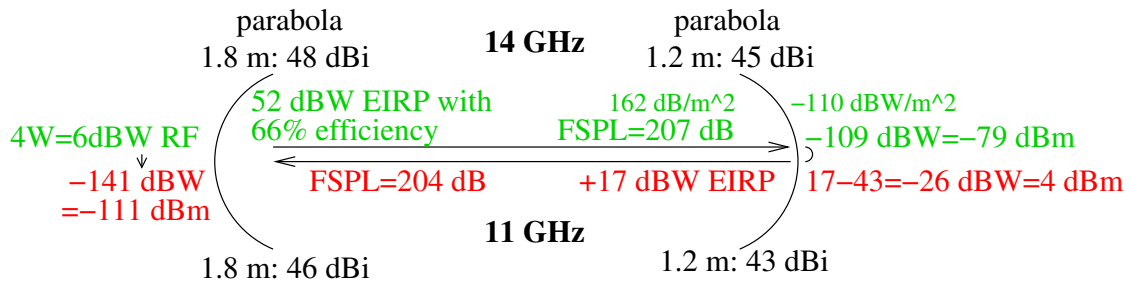
## References

- [1] K. Imamura & F. Takahashi, *Frequency and Time comparison – Two Way Time Transfer via a geostationary satellite*, J. of the Communication Research Laboratory **39** (1), 91–100 (1992)
- [2] Telesat, *Telstar 11N Technical Manual* (2008)

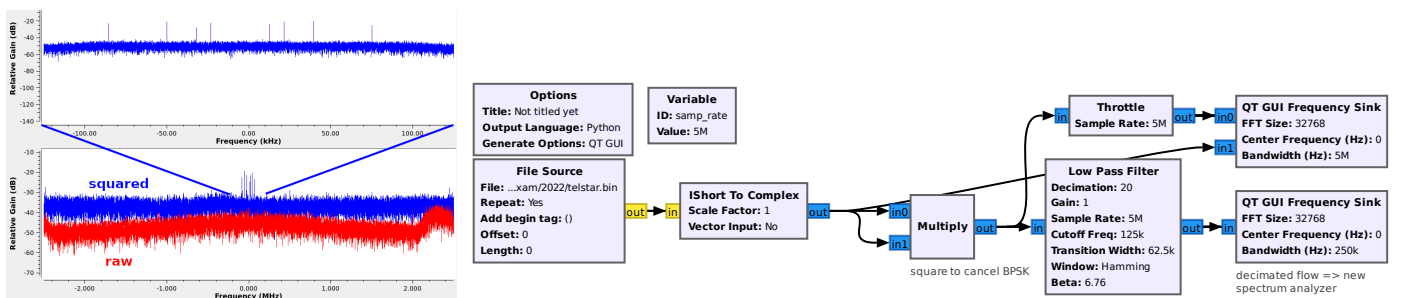
## Answers

1.  $\lambda = 300/14000 = 2.1$  cm and  $\lambda' = 300/11000 = 2.7$  cm
2. the  $D = 1.8$  m diameter antenna exhibits a gain of  $G_e = 10 \log_{10}(\pi^2 D^2 / \lambda^2) = 48$  dB when emitting and  $G_r = 10 \log_{10}(\pi^2 D^2 / \lambda'^2) = 46$  dB when receiving. The  $D' = 1.2$  m parabolic antenna on the satellite exhibits similarly a  $G'_r = 10 \log_{10}(\pi^2 D'^2 / \lambda^2) = 45$  dB when receiving and  $G'_e = 10 \log_{10}(\pi^2 D'^2 / \lambda'^2) = 43$  dB when transmitting.
3. 4 W is  $10 \log_{10} = 6$  dBW so accounting for antenna gain  $G_e$ , the radiated power is  $6 + 48 = 54$  dBW, close enough to the 52 dB provided in the article considering the efficiency of the antenna is e.g. 66%. The propagation of the 14 GHz signal over 36000 km (to be converted in meters in the Free Space Propagation Loss equation) is  $FSPL = 20 \log_{10}(14 \cdot 10^9) + 20 \log_{10}(30 \cdot 10^6) - 147.55 = 206.5$  dB close enough to the announced 207 dB. Thanks to the receiving antenna gain  $G'_r$ , the received power at the satellite is  $52 - 206.5 + 45 = -109.5$  dBW.
4. The power density at distance  $d = 36000$  km is the emitted power times the emitting antenna gain spread over the sphere of  $4\pi d^2$  so that in a  $1 \text{ m}^2$  area the power is  $52 - 207 = -155$  dBW/m<sup>2</sup> =  $-125$  dBm/m<sup>2</sup>, so that the power density is sufficiently low to avoid saturating the input stage of the amplifier. Two approaches for achieving this result is on the one hand as described in Fig. 2 (middle) where the  $1\text{-m}^2$  receiving antenna gain is included in Friis equation, or on the other hand the equivalent solution of replacing  $\lambda/(4\pi)$  in Friis equation with  $A = 1 \text{ m}^2$ . Both solution lead to the same solution since the  $1\text{-m}^2$  antenna gain is  $A/(\lambda^2/(4\pi))$
5. The received signal is frequency transposed and emitted back at a nominal power of 17 dBW=47 dBm EIRP. Considering the downlink antenna gain of 43 dBi, the RF power feeding the antenna is 4 dBm or 2.5 mW, well below the 100 W maximum output power.
6. The signal returned to the ground station is the transmitted radiated power minus free space propagation loss of 204.4 dB at 11 GHz plus receiving parabola gain at this same frequency or  $+17 - 204.4 + 46 = -141$  dBW =  $-111$  dBm.

As a summary of the link budget, with green the uplink and red the downlink, the folling figure summarizes the evolution of the radiofrequency power along its communication path:



7. The thermal noise floor of the parabola facing the sky is about  $k_B \times T = -174$  dBm/Hz so that in a 5 MHz bandwidth, the noise floor is  $-174 + 10 \log_{10}(5 \cdot 10^6) = -107$  dBm. The signal cannot be seen on the raw spectrum, similar to the case of GPS signals.
8. The situation is much worse in the case of the 60 cm dish since the ground station reception gain is now 37 dB instead of  $G'_r = 45$  dB. The received signal will be 8 dB lower due to missing antenna gain, and the chances of seeing the signal on a spectrum analyzer is even worse.
9. the length of the record times 200 ns/sample, inverse of 5 MHz. Even without loading the file, its size is enough to know the duration: 10485760 bytes with 2 bytes/integer and 2 integers (I, Q) in each sample so the nombre of samples is 2621440 at a rate of 5 MS/s so 0.52 s have been recorded.
10. See below (red curve): the Fourier transform of the signal does not exhibit any feature since power is still spread over the whole bandwidth by the BPSK modulation ...
11. ... but squaring the signal cancels the BPSK modulation and accumulates energy in each carrier at twice the frequency offset, allowing for the identification of communicating stations (blue curve). Zooming on the spectrum is achieved by **decimating after low pass filtering** to prevent aliasing.



12. a bias-T separates the DC component from the RF (AC) component. An inductor  $L$  prevents the radiofrequency wave from leaking in the power supply and a capacitor  $C$  prevents the DC supply voltage from damaging the radiofrequency amplifier on the receiver. The design considerations is for  $|Z_L| = L\omega$  to be large over  $|Z_C| = 1/(C\omega)$  at the operating angular frequency  $\omega = 2\pi f$  with  $f$  the microwave frequency, and  $|Z_C|$  to be small over the  $50\ \Omega$  characteristic impedance of the receiver. The capacitance will prevent the DC leaking in the amplifier whatever its value.
13.  $10 \times \log_{10}(10000) = 40$  dB SNR gain by **cross-correlating** the received signal with the known pseudo random code. Thanks to the correlation, the -111 dBm signal rises to -71 dBm, above the noise floor of -107 dBm as demonstrated experimentally here.
14. Even with the 60 cm parabola dish, the missing 8 dB will still allow the -79 dBm signal to be visible after squaring the signal.
15.  $1024\text{ MHz}/2048\text{ samples}=0.5\text{ MHz/bin}$
16. carrier at 100 MHz since only 50% modulation and two sidebands at  $100\text{ MHz}\pm 5\text{ kHz}$
17. 200 to 250 kHz
18.  $100\text{ km/h}$  is 27.8 ms inducing at 100 MHz a Doppler shift of  $100 \cdot 10^6 \frac{27.8}{3 \cdot 10^8} = 9\text{ Hz}$
19. This frequency shift is negligible over the signal bandwidth and will only induce a slight DC offset of the phase locked loop demodulator to be blocked by the capacitor located before the loudspeaker.
20. in the low SNR regime, thermal noise bandwidth cancels the numerator in the channel capacity and only the emitted power remains in the numerator.
21. no 137.5 MHz is allocated to satellite communication, while 434 MHz is the unlicensed ISM band.
22.  $300\text{ m}/\mu\text{s}$  is the speed of light so  $3\ \mu\text{s}$