Quartz tuning fork stroboscopic displacement field measurement

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slides and references at `jmfriedt.free.fr`

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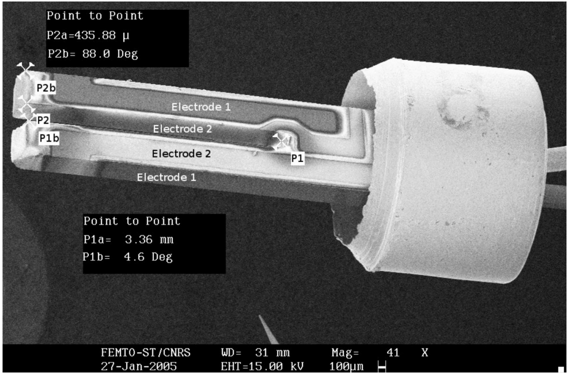
Quartz tuning fork



- the "quartz" in quartz watch
- the time reference is given by a tuning fork [1] [2] vibrating at $32768 = 2^{15}$ Hz $\Rightarrow$ counter (binary frequency divider) to reach 1 Hz
- piezoelectricity as an efficient way of generating mechanical vibration under control of an electrical signal
- acoustic signal: compact component since wave velocity is shrunk by 10^5 with respect to electromagnetic (10 km $\rightarrow$10 cm)
- initially a single beam vibrating at a few kHz: many spurious modes + lower quality factor
- 1960: replace a single beam with a tuning fork (Seiko Quartz Astron at 8192 Hz)
- from an engineering perspective: tuning fork = electrical dipole with extremely high quality factor
- from a physicist perspective: vibrating prongs made of a piezoelectric crystal

[1] W.E. Newell, *Miniaturization of tuning forks*, Science **161** (3848), 1320–1326 (1968)
[2] later an atomic transition from an atom in the Cs clock

- Tuning fork fundamental mode frequency

$$f_0 \simeq 0.1615 \frac{w}{L^2}\sqrt{\frac{E}{\rho}}$$

with length $L \simeq 3.36$ mm and width $w \simeq 436\ \mu$m, Young modulus $E \in 75 : 100$ GPa (anisotropic) and density $\rho \simeq 2650$ kg/m^3 – N.A: $f_0 \in [33 : 38]$ kHz

L. Bates *& al.*, *Determination of the temperature dependence of Young's modulus for stainless steel using a tuning fork*, J. Undergrad. Res. in Physics **18** (1), 9–13 (1999)

Note: J. Falter *& al.*. *Calibration of quartz tuning fork spring constants ...* (2014): the length of the prong should be to the stress minimum $>$ geometrical length

- high $Q \simeq 70000$ (drops to 7000 in air)
- Q is defined as the stored energy/energy dissipated during each oscillation
- the time needed to release the stored energy is Q/π oscillations or $Q/(\pi f)$ s @ $f = 32768$ Hz
- in an second order resonator circuit, the width at half height of the resonance (real part of admittance) is $\delta f = f/Q$
- low temperature coefficient

32.768kHz WATCH CRYSTAL, 6.2 x 02.1MM CYLINDER PACKAGE

AB26T

RoHS Compliant

FEATURES:
- Watch frequency
- 32.768kHz standard frequency

APPLICATIONS:
- Real time clock
- Measuring instruments
- Clock source for communication or A/V equipment

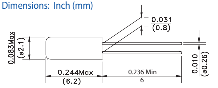
STANDARD SPECIFICATIONS:

PARAMETERS	
ABRACON P/N:	AB26T Series
Standard frequency:	**32.768kHz**
Additional frequencies available*	32.000kHz, 36.000kHz, 38.000kHz, 38.400kHz, 40.000kHz, 60.000kHz, 65.536kHz, 76.800kHz, 96.000kHz, 100.000kHz
Frequency range:	30kHz to 200kHz
Operating temperature:	-10°C to + 60°C (see option)
Storage temperature:	-40°C to + 85°C
Turn-over temperature:	+25°C ± 5°C
Frequency tolerance:	± 20 ppm max. for 32.768kHz (see option) ± 30 ppm max. for 30kHz ~ 200kHz (not including 32.768kHz)
Temperature Coefficient:	-0.034 ± 0.006 ppm/ T²
Equivalent series resistance:	35 kΩ max. (32.768kHz) 35 kΩ ~ 50 kΩmax. (30kHz ~ 200kHz)
Shunt capacitance C0:	0.8pF to 1.7pF typ.
Load capacitance CL:	12.5 pF typ. (see option)
Motional capacitance C1:	1 ~ 4 fF typ.
Capacitance ratio:	425 ~ 800 typ.
Quality factor:	70,000 typ. (32.768kHz)
Drive level:	1.0 μW max.
Aging @ 25° C first year:	± 3 ppm max. (32.768kHz) and ± 5 ppm max. (others)
Insulation resistance:	500 Mohms min. at 100Vdc ± 15V

* For additional frequencies please contact Abracon.

OUTLINE DRAWING:

Dimensions: Inch (mm)



OPTIONS & PART IDENTIFICATION: (Left blank if standard)

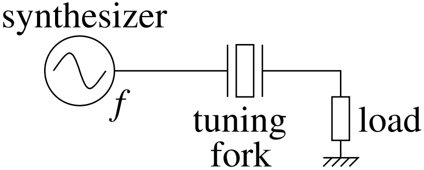
AB26T- Frequency- □ - □

CL Option
Please specify load cap. in pF (ex. 6pF)

Temperature options	
E	0°C to + 70°C
B	-20°C to + 70°C

Quartz tuning fork characterization

We wish to characterize the tuning fork electrical transfer function $Y(f)$ by sweeping f and monitoring the current (measured as voltage on a load resistor)



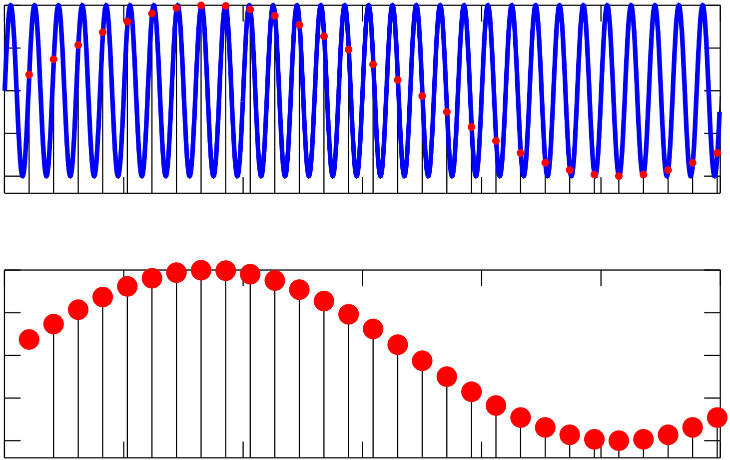
Questions:

1. what is the frequency step df at which f must be swept ?
2. how long shall we wait between two steps of df

⇒ challenge: observe the tuning fork prong motion using a personnal computer sound card and a commercial, off the shelf (COTS) webcam

Sound card

- Many current sound cards will exhibit >48 kS/s sampling rate (usually 96 kS/s or 192 kS/s)
- Driving the 32768 Hz tuning fork with this signal is possible ...
- ... but COTS webcams exhibit much lower framerate – 25 or 30 fps
- Can we "freeze" the prong motion during such long (40-33 ms) exposures ?
- Yes if the tuning fork is only illuminated when the prongs are in the same position: **stroboscopy**

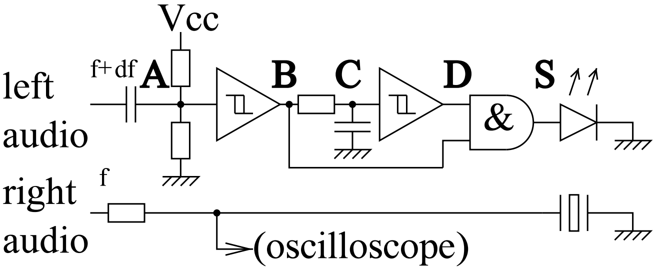


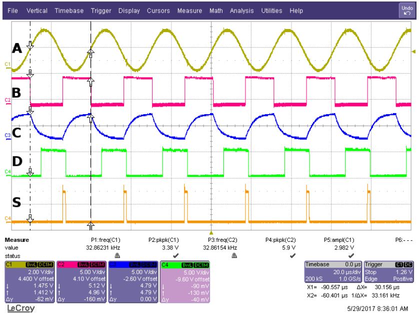
Stroboscopic setup

However, illuminating for a short duration (1/10th of the period is 3 μs or 300 kHz bandwidth) is way beyond the capability of a sound card

$\Rightarrow$ dedicated hardware controlled by a stereo sound card.

- Pulse = convert a sine wave to a square wave and to a pulse
- Pulse = phase shifted copies of the same sine wave
- Phase shift = time delay = RC circuit
- sine $\rightarrow$ Schmitt trigger (square, 2) $\rightarrow$ RC delay $\rightarrow$ Schmitt trigger (square) $\rightarrow$ & with (2) $\rightarrow$ pulse





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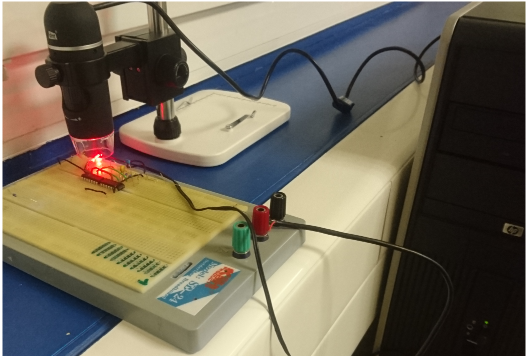
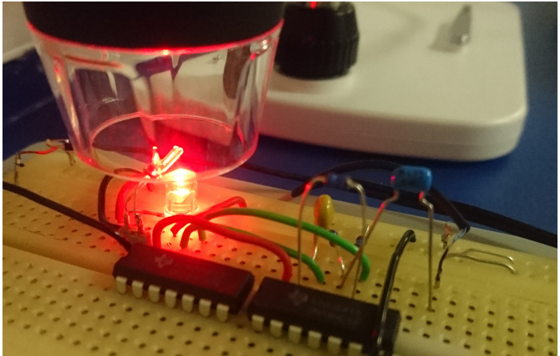
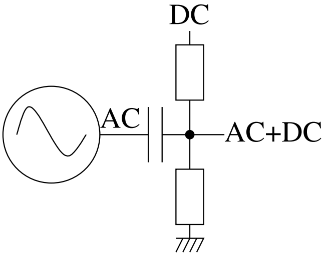




Hardware setup: the bias T

- a digital gate triggers around 1.5 V
- sound card output is 0-mean value
- offset the mean value without changing the AC spectral characteristics: bias T

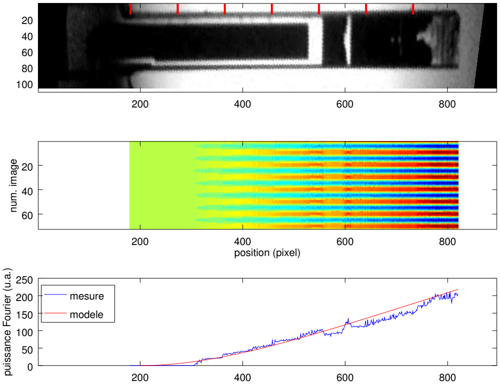


Question: considering the resistors have been selected as 1 kΩ resistances so that the current in the voltage divider bridge is $DC/2$ mA, how do we select the capacitor value so that the bias T output is the sum of DC+AC around 32768 Hz ?

Image processing

Movies (.avi format) have been recorded for various f driving frequencies

1. Decompose each movie to a series of pictures (`mplayer -vo jpeg movie.avi`)
2. Select an area representative of the prong motion
3. Compute the displacement of this area as the cross-correlation maximum position between the first image and the second image



4. Repeat for all images, cross-correlating the Nth image with the first
5. Display the motion of the prong
6. Since the motion is only a few pixels, we use an oversampling technique of fitting the correlation maximum and identifying the position of the fit maximum
7. Repeat for each frequency

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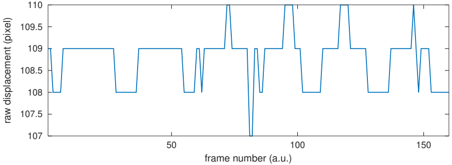

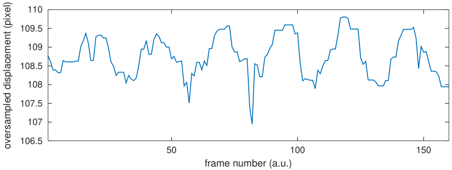

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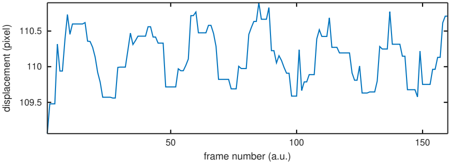

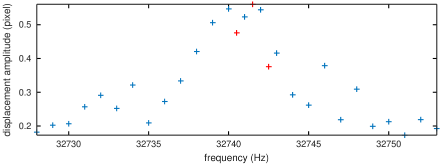

For more information ...

Tuning fork:

1. T. Hunkin & R. Garrod, *Secret Life Of Machines: The Quartz Watch* (1991), at `https://www.youtube.com/watch?v=nQ9_bOIj49s`
2. M.A. Lombardi, *The Evolution of Time Measurement, Part 2: Quartz Clocks*, IEEE Instrumentation & Measurement Magazine (Oct. 2011), pp.41–
3. J.-M Friedt, É. Carry, *Introduction au diapason à quartz*, Bull. de l'Union des Physiciens **879** 1137–1146 (2005) [in French]
4. J.-M Friedt, É. Carry, *Introduction to the quartz tuning fork*, American Journal of Physics, pp.415-422 (2007)
5. J.Marc, C. Canard, A. Vailly, V. Pichery, J.-M. Friedt, *Le diapason à quartz comme capteur : utilisation de la carte son de PC pour l'instrumentation*, Bull. de l'Union des Physiciens **958**, pp.1051–1073 (2013) [in French]
6. J.-M. Friedt, *Mesure stroboscopique du champ de déplacement d'un diapason à quartz au moyen d'une carte son et d'une webcam*, Bull. de l'Union des Physiciens **999** (2017) [in French]

Stroboscopy:

1. N.S. Gingrich, *Stroboscopic Aids in the Teaching of Physics*, American Journal of Physics **5**, 277 (1937)
2. S. Gupta & B. Jalali, *Time stretch enhanced recording oscilloscope* Appl. Phys. Lett. **94**, 041105 (2009)
3. J.S. Baskin & A.H. Zewail, *Freezing Atoms in Motion: Principles of Femtochemistry and Demonstration by Laser Stroboscopy*, J. Chem. Educ. **78** (6), p 737 (2001)

Software for image processing example

GNU/Octave version

```
pkg load signal
frequence=[32728:32753]; films=dir('./?????.avi');
for nfilm=1:length(films)
  system('rm -f ./*jpg');
  system(['mplayer -vo jpeg ./',films(nfilm).name]);
  d=dir('./*.jpg');
  x=imread(d(20).name);
%  figure(1);imagesc(x);
  x=x(:,:,1);
  reference=x(440,492:600);
  reference=reference-mean(reference);
  m=1;
  for k=21:180
    x=imread(d(k).name);
    x=x(:,:,1);
    mesure=x(440,492:600);
    mesure=mesure-mean(mesure);
    xc=xcorr(reference,mesure);
    [a(m),b(nfilm,m)]=max(xc);
    [u,v]=polyfit([b(nfilm,m)-2:b(nfilm,m)+2],xc([b(nfilm,m)-2:b(nfilm,m)+2]),2);
    xi=linspace(b(nfilm,m)-2,b(nfilm,m)+2,1024);
    yi=polyval(u,xi);
    [aa,bb]=max(yi); solution(m,nfilm)=xi(bb);
    m=m+1;
  end
%  figure(2);plot(solution-mean(solution));hold on
  amplitude(nfilm)=std(solution(:,nfilm))
end
```

Movies at `http://jmfriedt.org/TP_diapason.tar.gz`